MATH 1010 Sample Final Exam

1.) Solve the following equations.

a.) $1 + \sqrt{3x+1} = 5$

b.) $x^2 + 15 = 8x$

c.) $21 = 7 \cdot 4^x$

d.) $\log_5(3x) = 2$

2.) Solve the following system of equations. Write the answer as an ordered pair.

$$y = -4 - 2x$$
$$x + 3y = 3$$

3.) Let $f(x) = 2^x - 8$. Give the y-intercept as an ordered pair, and graph f.

4.) Graph the equation $2y + 5x = 10$.

5.) Simplify the following expression so that each variable occurs at most once and all exponents are positive. (Assume that all variables are positive.)

$$\frac{\left(x^{\frac{2}{5}}\right)^4 w^0}{x^{-\frac{7}{5}} y^6 y^2}$$

6.) Rewrite $x^2 + 6x + 10$ in completed square form.

7.) Let $f(x) = x^2 - 4x + 3$. Find the x-intercepts of f. What is the vertex of f? Write your answers as ordered pairs. Sketch the graph of f.

8.) Simplify $\sqrt{81x^8y^9}$ by removing all square factors from the square root. (Assume that all variables are positive.)

9.) Two functions are given: $f(x) = 3x - 1$ and $g(x) = x^2 + 4$.

a.) Find $(g \circ f)(x)$.

b.) Find $f^{-1}(x)$.

10.) The population of Ogden was 87,000 people in 2020. It is increasing by 1000 people every **two** years.

a.) Let $P(x)$ be the population of Ogden x years after 2020. Write the algebraic rule for $P(x)$.

b.) In which year will the population reach 90,000 people?

11.) A bicycle currently costs \$300. Due to inflation, its price is increasing by 8% per year. Let $f(x)$ be the price of bicycle x years from now. Select the type of function we can use to best model this situation.

◯ $f(x) = mx + b$

◯ $f(x) = ax^2 + bx + c$

◯ $f(x) = \sqrt{x}$

◯ $f(x) = a \cdot b^x$

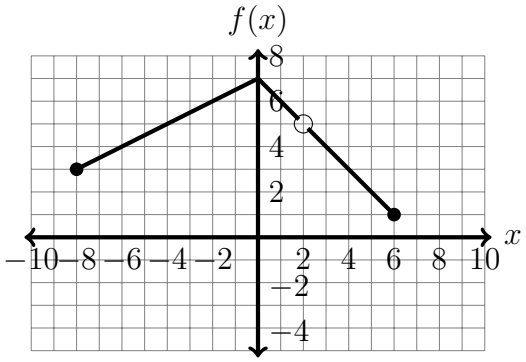
◯ $f(x) = \log_b(x)$

12.) A dog jumps into the air. Let $H(x) = -16x(x-2)$, where $H(x)$ represents the dog's height in feet and x represent the time in seconds after the dog jumps. Assume that the dog begins all jumps at the point $(0, 0)$.

a.) After the dog starts to jump, how many seconds does it take the dog to fall back to the ground? Give your answer in seconds.

b.) According to our model, what is the highest the dog jumps? Give your answer in)feet.

13.) Given the graph of the function f below, find the following.



a.) What is the domain of f? Write in interval notation.

b.) What is $f(0)$?

c.) Find the value(s) of x such that $f(x) = 5$.

Solutions to MATH 1010 Sample Final Exam

1.) a.) $\sqrt{3x+1} = 4$, so $3x+1 = 16$, and $3x = 15$.
Hence, $x = 5$.

b.) $x^2 - 8x + 15 = 0$, so

$$x = \frac{8 \pm \sqrt{8^2 - 4(15)}}{2} = \frac{8 \pm \sqrt{4}}{2} = 4 \pm 1$$

Hence, $x = 5$ or $x = 3$.

c.) $3 = 4^x$, so $\log_4(3) = x$.

d.) $\log_5(3x) = 2$, so $3x = 5^2 = 25$,
and $x = \frac{25}{3}$.

2.) $\left.\begin{aligned} y &= -4 - 2x \\ x + 3y &= 3 \end{aligned}\right\} \Rightarrow x + 3(-4-2x) = 3$

$\Rightarrow x - 12 - 6x = 3$

$\Rightarrow -5x = 15$

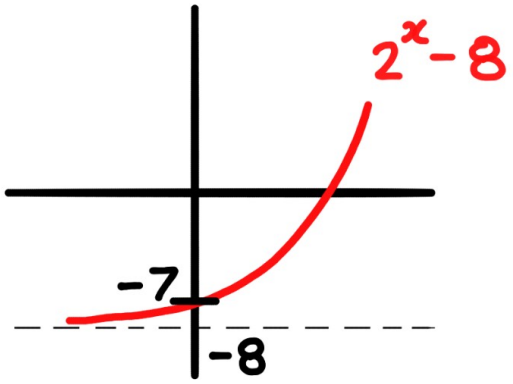
$\Rightarrow x = -3$

$\left.\begin{aligned} x &= -3 \\ y &= -4 - 2x \end{aligned}\right\} \Rightarrow y = -4 - 2(-3) = 2.$

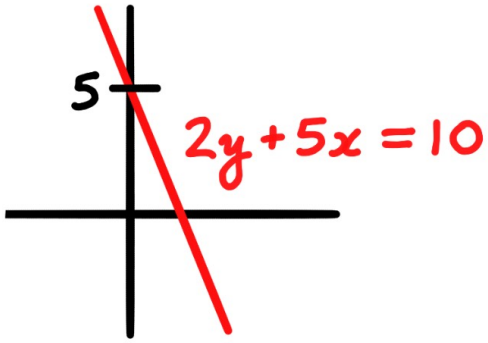
Thus, $(x,y) = (-3,2)$.

3.) y-intercept is where $x=0$, so

$f(0) = 2^0 - 8 = 1 - 8 = -7$. As an ordered pair, $(x,y) = (0,-7)$.



4.) $2y = 10 - 5x$, so $y = 5 - \frac{5}{2}x$. Straight line, slope $-\frac{5}{2}$, y-intercept 5.



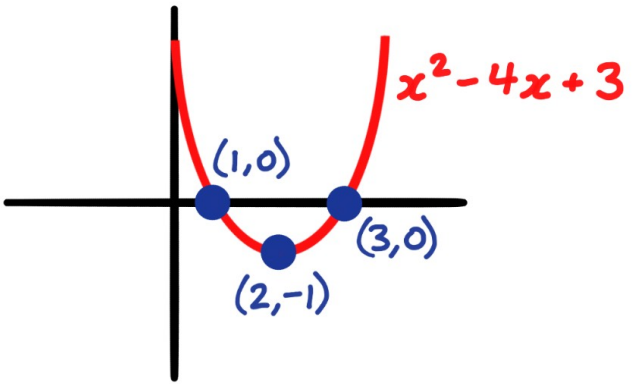
5.) $$\frac{x^{8/5} \cdot 1}{x^{-7/5} \cdot y^8} = \frac{x^{15/5}}{y^8} = \frac{x^3}{y^8}$$

6.) $$x^2 + 6x + 10 = \left(x + \tfrac{1}{2}6\right)^2 + 10 - \frac{6^2}{4}$$
$$= (x+3)^2 + 10 - \frac{36}{4}$$
$$= (x+3)^2 + 1$$

7.) Quadratic formula gives the roots of x^2-4x+3 as

$$x=\frac{4\pm\sqrt{4^2-4(3)}}{2}=\frac{4\pm\sqrt{4}}{2}=2\pm1$$

so $x=3$ and $x=1$ are the roots and x-intercepts: as ordered pairs, $(3,0)$ and $(1,0)$. The vertex is half-way between the roots, so at $x=2$. There, $y=2^2-4(2)+3=-1$, so $(2,-1)$ is the vertex.



8.) $\sqrt{81\,x^8\,y^8}\sqrt{y}=9x^4y^4\sqrt{y}$

9.) a.) $gof(x)=(3x-1)^2+4=9x^2-6x+5$

b.) $x=3f^{-1}(x)-1$ so $x+1=3f^{-1}(x)$ and $f^{-1}(x)=\frac{x+1}{3}$.

10.) Population increases by 500 each year.

a.) $P(x) = 87{,}000 + 500x$

b.) $90{,}000 = 87{,}000 + 500x$, so

$3{,}000 = 500x$ and $x = \frac{3{,}000}{500} = 6$

Year is $2020 + 6 = 2026$.

11.) $f(x) = a \cdot b^x$ is exponential growth.

For this problem, $f(x) = 300 \cdot (1.08)^x$.

12.) a.) height of $0 = -16x(x-2)$ means either $x = 0$ (when the jump occurs) or $x = 2$ (when dog lands). Answer is 2 seconds.

b.) vertex is at $x = 1$, where

$H(1) = -16(1-2) = 16$ feet.

13.) a.) Domain is $[-8, 2) \cup (2, 6]$.

b.) $f(0) = 7$

c.) $x = -4$ only. ($x = 2$ is not in the domain.)