University of Utah, Department of Mathematics
August 2023, Algebra II Qualifying Exam

There are five problems on the exam. You may attempt as many problems as you wish; three correct solutions count as a high pass. 2 correct solutions count as a pass. Show all your work, and provide reasonable justification for your answers.

1. Prove that the alternating group A_5 is simple.
2. Find all the groups G of order 18.
3. How many irreducible polynomials of degree 12 in $\mathbb{F}_p[x]$ are there?
4. Prove that the Galois group of an irreducible polynomial $f(x) \in \mathbb{Q}[x]$ of degree p (a prime) with one pair of complex conjugate roots and $p-2$ real roots is the full symmetric group S_p.
5. Find the Galois group of $x^8 - 2 \in \mathbb{Q}[x]$. In particular, prove that it has order 16 and is **not** the dihedral group.