MATH 6320 QUAL, AUGUST 2025

[YOUR NAME:]

1) Let p be an odd prime. Prove that there are only two non isomorphic groups of order $2p$.

2) Let $\mathbb{Q}$ be the set of rational numbers, considered a group with respect to the usual addition. Prove that $\mathbb{Q}$ is indecomposable, that is, it is not isomorphic to a direct sum $A \oplus B$ of two non-trivial subgroups.

3) Let $\zeta_8 = \exp(2\pi i/8)$, complex eight root of one. Use Galois theory to determine all subfields of $\mathbb{Q}(\zeta_8)$.

4) Let F be a field of characteristic p. Let S_n be the group of permutations of $X = \{1, 2, \ldots, n\}$. Let $V = F[X]$ be the space of F-valued functions. It is a linear representation of S_n, where $\sigma \in S_n$ acts on $f \in F[X] \cong F^n$ by $(\pi(\sigma)f)(x) = f(\sigma^{-1}x)$. We have an intertwining map $P : F[X] \to F$, where on F we have the trivial action of S_n, given by $f \mapsto f(1) + f(2) + \cdots + f(n)$. Prove that this map splits, that is, there exists an intertwining map Q in the opposite direction such that $P \circ Q = 1$, if and only if p does not divide n.