DEPARTMENT OF MATHEMATICS
University of Utah
Ph.D. PRELIMINARY EXAMINATION IN COMPLEX ANALYSIS
August 2020

Instructions: This exam has two sections, A and B. Three (3) completely correct problems in section A will be a high pass and any three (3) completely correct problems is a passing exam. Note that solutions for problems in section B will not be counted towards a high pass. Be sure to provide all relevant definitions and statements of theorems cited.

A

1. Let $f_i : \mathbb{D} \to \mathbb{C}$ be a sequence of injective holomorphic functions so that $f_i \to f_\infty$ uniformly on compact sets. If f_∞ is not injective, what can you say about it? Prove this.

2. Prove that if $f : \{z : 1 < |z| < 2\} \to \mathbb{C}$ is holomorphic then $f(z) = \sum_{j=-\infty}^{\infty} a_j z^j$. What can you say about the power series $\sum_{j=0}^{\infty} a_j z^j$ and $\sum_{j=-\infty}^{0} a_j z^{-j}$? Prove this.

3. Prove or disprove: An entire function f so that $f(n) = 0$ for all $n \in \mathbb{Z}$ is identically 0.

4. Let $f : \mathbb{H} \to \mathbb{D}$ be a holomorphic function so that $f(i) = 0$. What can you say about $|f(2i)|$? (Of course with proof.)

5. Show that if $f : \{z : 0 < |z| < 1\} \to \mathbb{C}$ is meromorphic and has poles at $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$ then the range of f is dense in $\mathbb{C}$.

B

1. State the Cauchy Riemann equations and prove that a function is holomorphic on an open set iff it satisfies the Cauchy Riemann equations.

2. Use the methods of complex analysis to compute $\int_{-\infty}^{\infty} \frac{dx}{(1+x^2)^3}$.