MATH 1010 ~ Intermediate Algebra

Chapter 2: LINEAR EQUATIONS AND INEQUALITIES

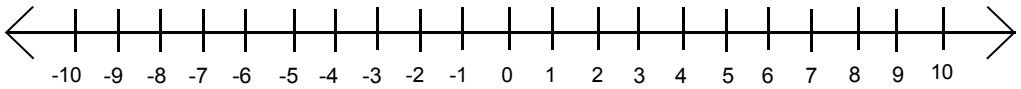
Section 2.5: Absolute Value Equations and Inequalities

Objectives:

- Solve absolute value equations
- Solve inequalities involving absolute value.

Sketch the solution on the number line.

$|2x-3| < 5$



$|x| = a$ means x is a units away from 0 on the $\mathbb{R}$ number line

① EXAMPLE:

Defn

$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$

$|-3| = 3 = -(-3)$

a) $|x| = 5$

$x = 5, -5 \quad (x = \pm 5)$

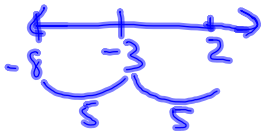
b) $|x + 3| = 5$

① ②

$x+3=5 \qquad x+3=-5$

$x=2 \qquad x=-8$

$\rightarrow |x-(-3)|=5$



c) $|3x - 2| = 8$

① ②

$3x-2=8 \qquad 3x-2=-8$

$3x=10 \qquad 3x=-6$

$x=10/3 \qquad x=-2$

$\Rightarrow x=-2, 10/3$

d) $|2x - 1| + 7 = -10$

$-7 \quad -7$

$|2x-1| = -17 \Rightarrow$ N.S.

e) $|3x - 4| = |7x - 16|$

cases

① (a) $+ \; +$ (b) $- \; -$

② (c) $+ \; -$ (d) $- \; +$

① ②

$3x-4=7x-16 \qquad -(3x-4)=7x-16$

$-3x \quad -3x \qquad -3x+4=7x-16$

$-4=4x-16 \qquad +3x \quad +3x$

$12=4x \qquad 4=10x-16$

$3=x \qquad 20=10x$

$2=x$

f) $|x + 2| = |x + 9|$

① ②

$x+2=x+9 \qquad x+2=-(x+9)$

$-x \quad -x \qquad x+2=-x-9$

$2 \neq 9 \qquad 2x+2=-9$

N.S. $\qquad 2x=-11$

$x=-11/2$

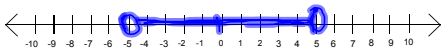
ABSOLUTE VALUE INEQUALITIES

|x| < a means

all x-values that are less than a units away from zero

|x| < 5 and

$-5 < x < 5$

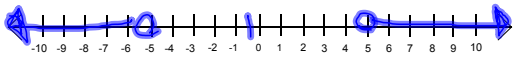


|x| > a means

all x-values greater than a units away from zero

|x| > 5 OR

$x > 5$ or $x < -5$



$|x| < 2$

$x < 2$ $\quad -x < 2$

$-2 < x < 2$ $\quad x > -2$

② Examples

a) $|x - 4| \leq 6$ and

① $x - 4 \leq 6$ $\quad -(x-4) \leq 6$

$x \leq 10$ $\quad \div -1$

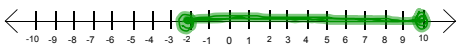
$x - 4 \geq -6$

$x \geq -2$

$-2 \leq x \leq 10$

② $-6 \leq x - 4 \leq 6$

$-2 \leq x \leq 10$



b) $|3x - 4| \geq 5$ OR

$3x - 4 \geq 5$ $\quad -(3x-4) \geq 5$

$+4 \quad +4$ $\quad 3x - 4 \leq -5$

$3x \geq 9$ $\quad +4 \quad +4$

$x \geq 3$ $\quad 3x \leq -1$

DR $\quad x \leq -1/3$

Watch for this: $|2x + 3| \leq -2$ N.S.

abs. value of something ≤ negative # never happens

$|2x+3| \geq -2$

x can be any ℝ