MATH 1010 ~ Intermediate Algebra Chapter 3: GRAPHS AND FUNCTIONS

Section 3.3: Slope and Graphs of Linear Equations

Objectives:

- Determine the slope of a line through two points.
- Graph linear equations in slope-intercept form.
- Use slopes to determine whether two lines are parallel, perpendicular or neither.
- Use slopes to describe rates of change in real-life problems.

aka slope

Positive or negative slope?

Parallel or perpendicular?

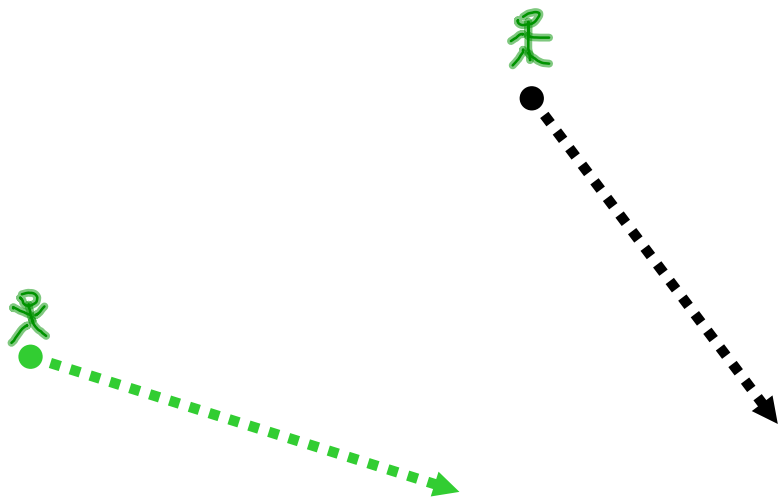
$3x - 2y = 6$

$3x + 2y = 4$

$2x - 3y = -6$

$2x + 3y = -2$

Slope is something you are familiar with. Think of a ski hill. What makes it more exciting?



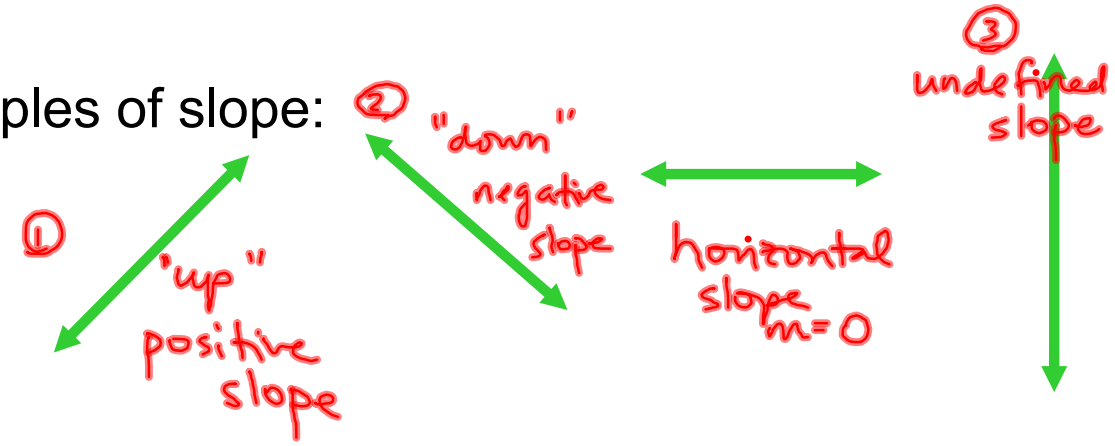
The slope of a line given points (x_1,y_1) and (x_2,y_2) is

$$\frac{y_1 - y_2}{x_1 - x_2} = m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{rise}}{\text{run}} = \frac{\text{change in y}}{\text{change in x}}$$

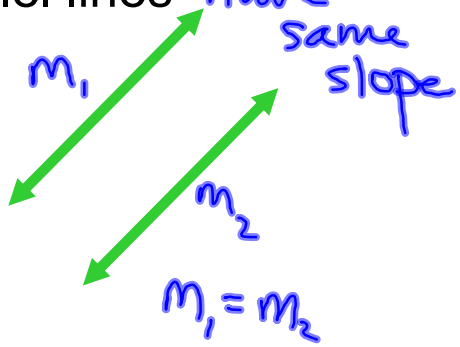
memorize

vert / horiz.

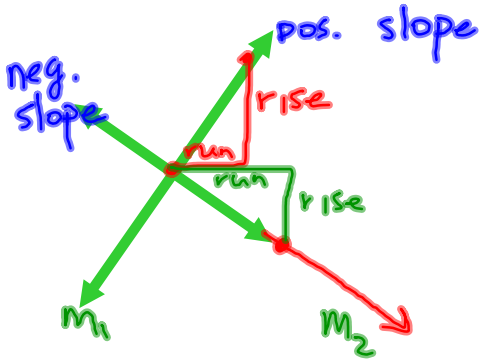
Examples of slope:



Parallel lines have same slope



Perpendicular lines



$m_1 m_2 = -1$

$m_1 = \frac{-1}{m_2}$

① EXAMPLE

Find the slope of the line connecting each pair of points.

$m = \frac{y_2 - y_1}{x_2 - x_1}$ (x_1, y_1) (x_2, y_2)

a) (-3,2) and (5,2)

$m = \frac{2-2}{5-(-3)} = \frac{0}{8} = 0$

b) (-3,2) and (-3,5)

$m = \frac{5-2}{-3-(-3)} = \frac{3}{0}$ undefined (vertical line)

c) (-3/2, -1/2) and (5/8,1/2)

$m = \frac{\frac{1}{2} - \left(-\frac{1}{2}\right)}{\frac{5}{8} - \frac{-3}{2}} = \frac{1}{\frac{5}{8} + \frac{3}{2}\left(\frac{4}{4}\right)} = \frac{1}{\frac{5}{8} + \frac{12}{8}}$

$= \frac{1}{17/8} = \frac{8}{17}$

$1 \div \frac{17}{8} = 1 \cdot \frac{8}{17}$

d) (9,-2) and (-7,2)

$m = \frac{2-(-2)}{-7-9} = \frac{4}{-16} = \frac{-1}{4}$

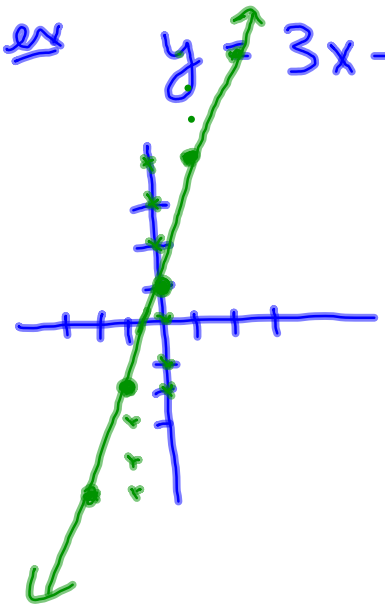
How to graph the equation of a line in slope-intercept form:

Slope-intercept form: $y = mx + b$

m = slope (0,b) is the y-intercept. (not: b is not the y-intercept)

ex $y = 3x + 1$

$m = 3 = \frac{3}{1} = \frac{-3}{-1}$



② EXAMPLE

Sketch each of these.

a) $y = 3x + 2$

$m = 3 = \frac{3}{1} = \frac{-3}{-1}$ (0, 2)

b) $3x - 2y = 8$

(0, -4) $m = \frac{3}{2}$

$-2y = -3x + 8$

$y = \frac{3}{2}x - 4$

c) $6y - 5x + 15 = 0$

$6y = 5x - 15$

$y = \frac{5}{6}x - \frac{15}{6}$

$m = 5/6$ $(0, -5/2)$

$y = \frac{5}{6}x - \frac{5}{2}$

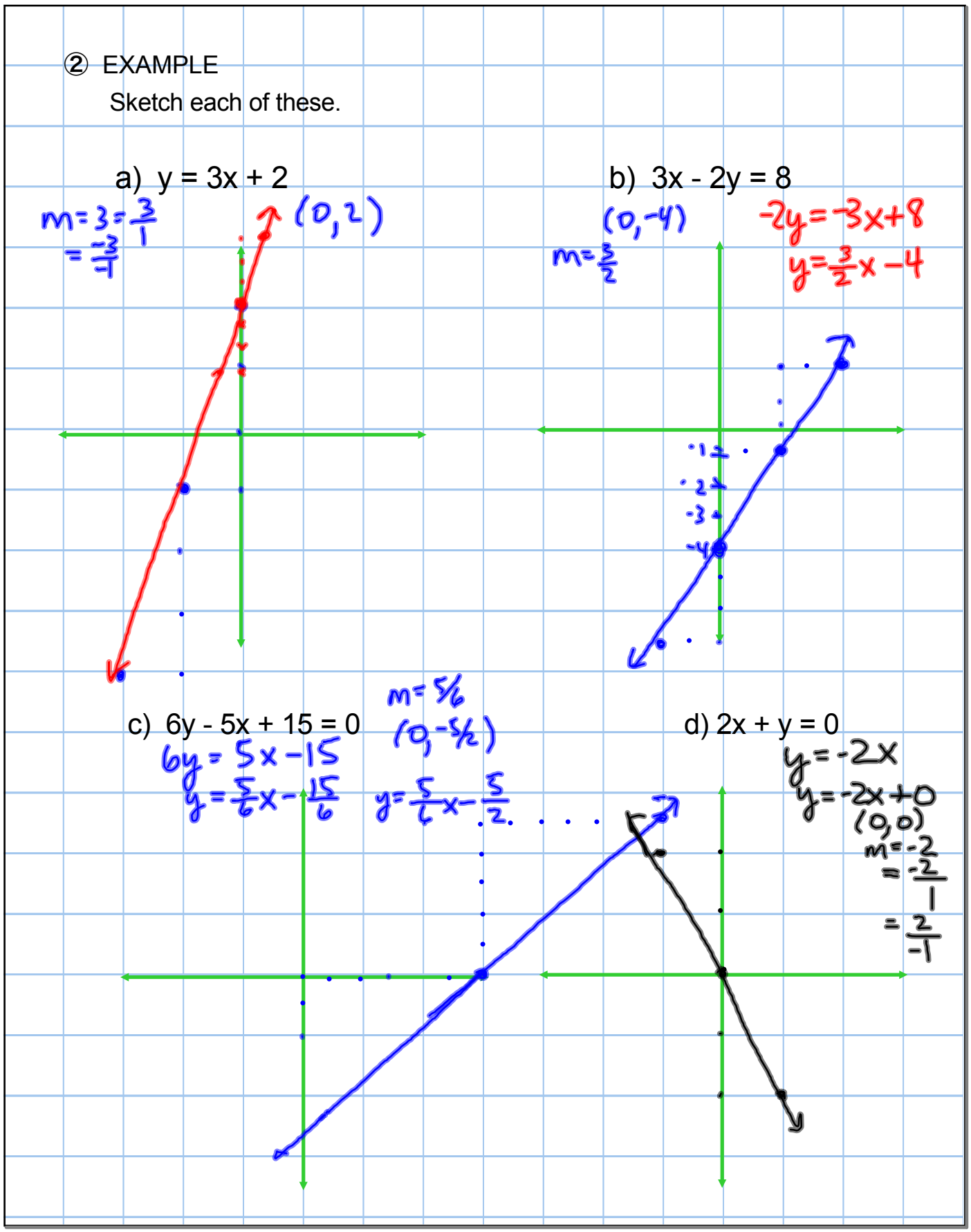
d) $2x + y = 0$

$y = -2x$

$y = -2x + 0$

(0, 0)

$m = -2 = \frac{-2}{1} = \frac{2}{-1}$



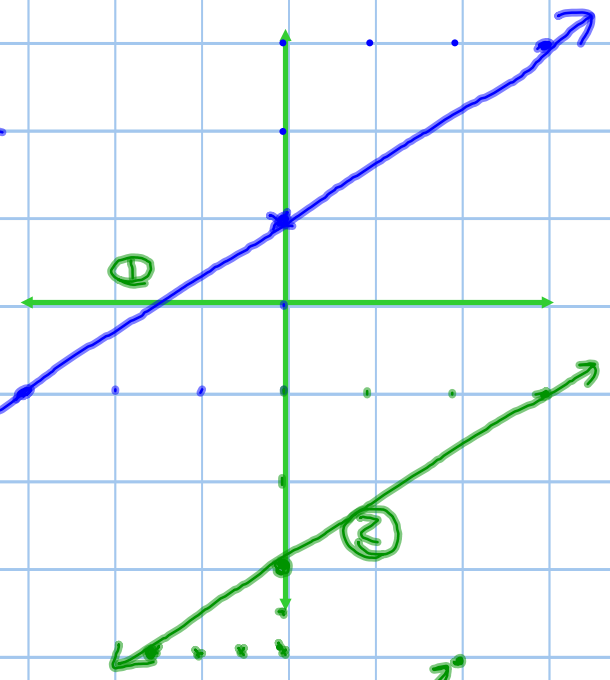
Parallel and Perpendicular lines

Parallel lines:

$m = \frac{2}{3} = \frac{-2}{-3}$

parallel $y = \frac{2}{3}x + 1$ ① y-int (0,1)

$y = \frac{2}{3}x - 3$ ② y-int (0,-3)

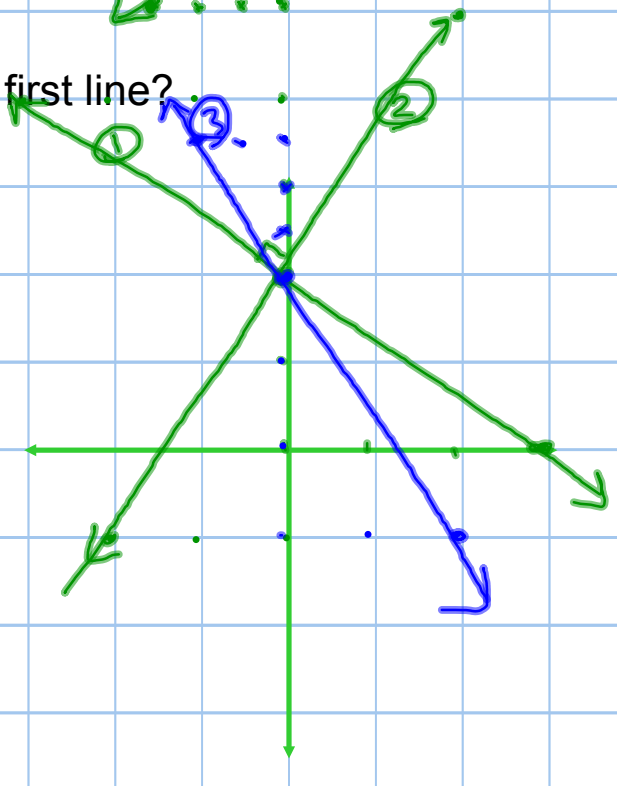


Which will be perpendicular to the first line?

$y = -\frac{2}{3}x + 2$ ① $m = -\frac{2}{3}$

⊥ $y = \frac{3}{2}x + 2$ ② $m_{\perp} = \frac{3}{2}$

$y = -\frac{3}{2}x + 2$



③ EXAMPLE

Given the line $2x - 3y = 6$

$-2x \quad -2x$

$-3y = -2x + 6$

$y = \frac{2}{3}x - 2 \Rightarrow m = \frac{2}{3}$

a) Write an equation of a line parallel to that line through the origin, (0,0).

(The form will be y = mx + 0)

$y = mx$

$y = \frac{2}{3}x$

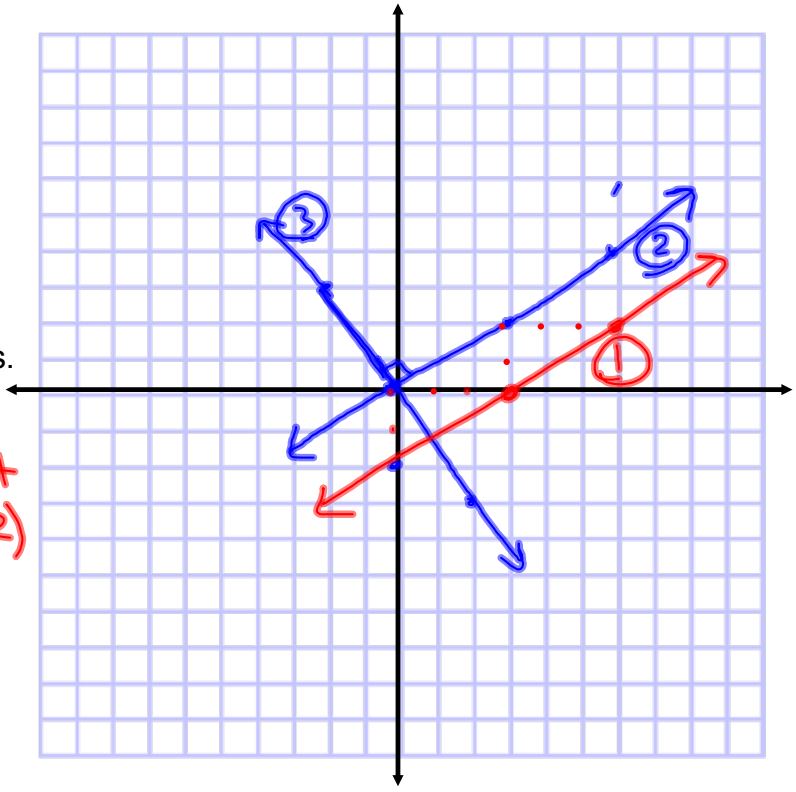
b) Write an equation of a line perpendicular to that line through the origin,(0,0).

(The form will be y = mx + 0)

$m_{\perp} = \frac{-3}{2}$

$\perp \quad y = \frac{-3}{2}x$

c) Graph the three lines.



① $y = \frac{2}{3}x - 2$ y-int (0,-2)

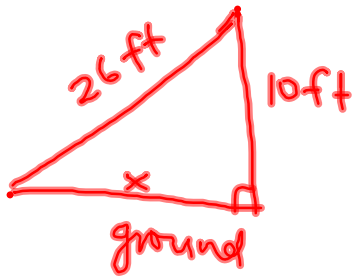
∥ ② $y = \frac{2}{3}x$

$\perp$ ③ $y = \frac{-3}{2}x$

④ EXAMPLE:

Slope in real life ~ make a sketch of the problem.

a) A skate-board ramp has a length of 26 feet. The top is 10 feet off the ground. What is the slope of the ramp?

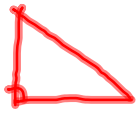


$x^2 + 10^2 = 26^2$

$x^2 + 100 = 676$

$x^2 = 576$

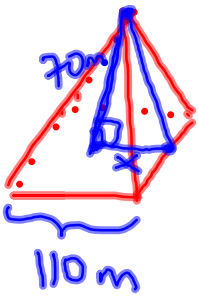
$x = \sqrt{576} = 24$ ft



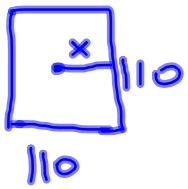
$\Rightarrow$ slope $= \frac{10}{24} = \frac{5}{12}$

b) One of the Egyptian Pyramids is 70 meters tall and 110 meters across the base. What is the slope of the face of the pyramid?

(pyramid w/ square base)



base



$x = \frac{1}{2}(110) = 55$

$\Rightarrow$ slope $= \frac{70}{55} = \frac{14}{11}$