Section 4.2: LINEAR SYSTEMS IN TWO VARIABLES

Objectives:

- Solve systems of equations by elimination.
- Use systems of equations to solve real life problems.

3 drinks + 4 doughnuts = $10.00
2 drinks + 2 doughnuts = $ 6.00

How much is 1 doughnut?

The method of elimination

1. Obtain coefficients for x (or y) that are opposites by multiplying all terms of one or both equations by suitable non-zero constants.

2. Add the equations to eliminate one variable and solve the resulting equation for the remaining variable.

3. Back-substitute the value obtained in step 2 in either of the original equations and solve for the other variable.

4. Check your solution in both of the original equations.

step 1

① $4x - 5y = 13$
② $-5(\; 3x - y = 7\;)-5$

$\iff$ ① $4x - 5y = 13$
② $-15x + 5y = -35$

step2 +

$-11x = -22$ resulting eqn

$x = 2$

step 3

① $4(2) - 5y = 13$

$8 - 5y = 13$
$-8 \qquad\quad -8$

$-5y = 5$

$y = -1$

$\boxed{\text{soln: } (2, -1)}$

(choose to get rid of x)

① $2(\; 3x + 9y = 8\;)$
② $-3(\; 2x + 6y = 7\;)$

$\iff$ ① $6x + 18y = 16$
$+$ ② $-6x - 18y = -21$

$0 \neq -5$

$\Rightarrow$ N.S.
(parallel lines)

① EXAMPLE:

Solve these systems by elimination.

① a) $-x + 2y = 9$
② $x + 3y = 16$
$+$

$$5y = 25$$
$$y = 5$$

② $x + 3(5) = 16$
$x + 15 = 16$
$x = 1$

$$\boxed{\text{soln: } (1, 5)}$$

① b) $3y = 2x + 21$
② $\frac{2}{3}x = 50 + y$

$\Leftrightarrow$ ① $-2x + 3y = 21$
② $3\left(\frac{2}{3}x - y\right) = (50)3$

$\Leftrightarrow$ ① $-2x + 3y = 21$
② $2x - 3y = 150$
$+$

$$0 \neq 171$$

$\Rightarrow$ N.S.

(parallel lines)

-2 c) $(4x = 6 + 5y)$
$8x = 12 + 10y$

$\Leftrightarrow$ $-8x = -12 + -10y$
$+ \; 8x = 12 + 10y$

$$0 = 0 \quad \text{(true)}$$

$\Rightarrow$ same lines
(every pt is the same)

② EXAMPLE:

Solve these applications by an appropriate method.

a) An SUV costs $26,445 and an average of $0.18 per mile to maintain. A hybrid model of the SUV costs $31,910 and $0.13 to maintain.

After how many miles will the cost of the SUV exceed the cost of the hybrid?

cost of SUV: ① $26,445 + 0.18x = y$

cost of hybrid: ② $31910 + 0.13x = y$

x = # miles
y = cost ($)

$x = ?$ when costs are equal

$$\begin{array}{rl} ① & -26445 - 0.18x = -y \\ + ② & 31910 + 0.13x = y \\ \hline & 5465 - 0.05x = 0 \end{array}$$

$$\frac{5465}{0.05} = \frac{0.05x}{0.05} \Rightarrow x = 109{,}300 \text{ miles}$$

b) A total of $1790 was made by selling 200 adult tickets and 316 children's tickets to a charity event. The next night a total of $937.50 was made by selling 100 adult tickets and 175 children's tickets.

Find the price of each type of ticket.

c = price of child ticket
a = price of adult ticket

① $1790 = 200a + 316c$

② $-2(937.5 = 100a + 175c)$

$$\begin{array}{rl} \Leftrightarrow & 1790 = 200a + 316c \\ + & -1875 = -200a - 350c \\ \hline & -85 = -34c \end{array}$$

$\frac{-85}{-34} = c$

$c = \$2.50$

①

$1790 = 200a + 316(2.5)$

$1790 = 200a + 790$

$-790 \qquad -790$

$1000 = 200a$

$\boxed{a = \$5}$