MATH 1010 ~ Intermediate Algebra

Chapter 8: QUADRATIC EQUATIONS AND FUNCTIONS

Chapter 8: Four Strategies for Solving Quadratic Equations

Objectives:

- ✯ Solve quadratic equations by factoring.
- ✯ Solve quadratic equations by the Square Root Property.
- ✯ Solve quadratic equations by completing the square.
- ✯ Solve quadratic equations using the quadratic formula.

$$3x^2 - 2x - 5 = 0$$

Strategy 1: Solve by factoring. (works sometimes)
only when quadratic is factorable

a) $x^2 - x - 6 = 0$

$(x-3)(x+2)=0$

$x-3=0 \qquad x+2=0$

$x=3 \qquad x=-2$

1. everything is on one side of =, with 0 on other side
2. factor quadratic
3. Set each factor equal to 0 and solve.

b) $8x^2 - 10x + 3 = 0$

$8 \cdot 3 = 24$

$-4 \cdot -6$

	$4x$	-3
$2x$	$8x^2$	$-6x$
-1	$-4x$	3

$(4x-3)(2x-1)=0$

$4x-3=0 \qquad 2x-1=0$

$4x=3 \qquad 2x=1$

$x=3/4 \qquad x=1/2$

Strategy 2: Take the square root of both sides.

(only works sometimes)

★ when you have only one instance of variable

a) $2x^2 = 14$

$\frac{2x^2}{2} = \frac{14}{2}$

$\sqrt{x^2} = \pm\sqrt{7}$

$x = \sqrt{7}, -\sqrt{7}$

b) $(2x-5)^2 - 3 = 0$

$\sqrt{(2x-5)^2} = \pm\sqrt{3}$

$2x-5 = \pm\sqrt{3}$

$2x = 5 \pm \sqrt{3}$

$x = \frac{5 \pm \sqrt{3}}{2}$

c) $x^2 + 9 = 0$

$\sqrt{x^2} = \pm\sqrt{-9}$

$x = \pm\sqrt{-9} = \pm\sqrt{-1}\sqrt{9}$

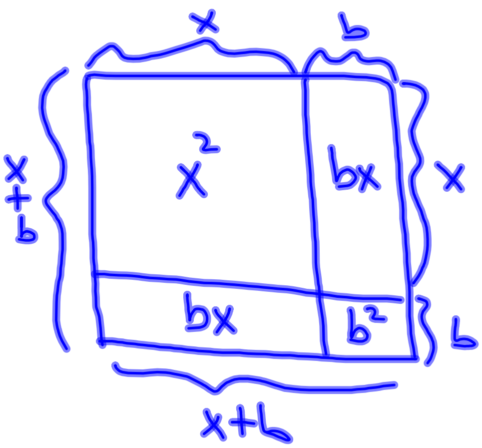
$\boxed{x = \pm 3i}$

- peeling off the layers of ops that have happened to the variable
- NOTE: when take $\sqrt{\ }$ of both sides of an eqn, must consider $\pm$ possibilities

NOTE: There will always be two solns to quadratic eqns – ① they might be complex (if so, the solns are complex conjugates); ② the two solns might be the same (one soln repeated).

Strategy 3: Complete the square.

(works always)



$$\text{Area} = (x+b)^2$$
$$= x^2 + 2bx + b^2$$

Algebraic strategy

① notice quadratic doesn't factor

② factor out leading coefficient from x^2 and x terms; move constant to other side of = .

③ add missing term (inside ( )) to both sides
missing term: $\left(\frac{1}{2} \cdot \text{coefficient of } x\right)^2$

④ factor & continue solving

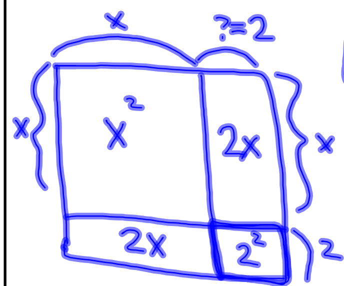
a) $(x^2 + 4x) + 1 = 0$

$x^2 + 4x = -1$

$x^2 + 4x + 4 = -1 + 4$

$(x+2)^2 = 3 \Rightarrow x + 2 = \pm\sqrt{3}$

$\boxed{x = -2 \pm \sqrt{3}}$



b) $x^2 - 10x - 15 = 0$

$x^2 - 10x = 15$

$x^2 - 10x + 25 = 15 + 25$

$\left(\frac{-10}{2}\right)^2 = 25$

$(x-5)^2 = 40$

$x - 5 = \pm\sqrt{40}$

$x = 5 \pm \sqrt{40}$

$x = 5 \pm \sqrt{4}\sqrt{10}$

$\boxed{x = 5 \pm 2\sqrt{10}}$

c) $3x^2 - 24x - 5 = 0$

$3(x^2 - 8x) - 5 = 0$

$3(x^2 - 8x + 16) = 5 + 48$

missing term: $\left(-\frac{8}{2}\right)^2 = 16$

(or $(x^2 - 8x + 16) = \frac{5}{3} + 16$)

$\frac{3(x-4)^2}{3} = \frac{53}{3}$

$(x-4)^2 = \frac{53}{3}$

$x - 4 = \pm\sqrt{\frac{53}{3}}$

$\boxed{x = 4 \pm \sqrt{\frac{53}{3}}}$

d) $x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$

$x = 3, -2$

Completing square:

$x^2 - x = 6$

$x^2 - x + \frac{1}{4} = 6 + \frac{1}{4}$

missing term: $\left(\frac{-1}{2}\right)^2 = \frac{1}{4}$

$\left(x - \frac{1}{2}\right)^2 = \frac{25}{4}$

$x - \frac{1}{2} = \pm\sqrt{\frac{25}{4}}$

$x = \frac{1}{2} \pm \frac{5}{2} = \frac{1}{2} + \frac{5}{2}, \frac{1}{2} - \frac{5}{2}$

$= 3, -2$

Strategy 4: Use the Quadratic Formula.

(always works)

Complete the square:

$a,b,c \in \mathbb{R}$

$$ax^2+bx+c=0$$
$$ax^2+bx=-c$$
$$a\left(x^2+\frac{b}{a}x\right)=-c$$

missing term:

$$\left(\frac{b}{a}\cdot\frac{1}{2}\right)^2=\left(\frac{b}{2a}\right)^2=\frac{b^2}{4a^2}$$

$$\left(x^2+\frac{b}{a}x\right)=-c/a$$
$$\left(x^2+\frac{b}{a}x+\frac{b^2}{4a^2}\right)=\frac{-c}{a}+\frac{b^2}{4a^2}$$

check

$$\left(x+\frac{b}{2a}\right)^2$$
$$=\left(x+\frac{b}{2a}\right)\left(x+\frac{b}{2a}\right)$$
$$=x^2+\frac{b}{2a}x+\frac{b}{2a}x+\frac{b^2}{4a^2}$$
$$=x^2+\frac{b}{a}x+\frac{b^2}{4a^2} \checkmark$$

$$\left(x+\frac{b}{2a}\right)^2=\frac{-4ac}{4a^2}+\frac{b^2}{4a^2}$$
$$\left(x+\frac{b}{2a}\right)^2=\frac{b^2-4ac}{4a^2}$$
$$x+\frac{b}{2a}=\pm\sqrt{\frac{b^2-4ac}{4a^2}}$$
$$x=\frac{-b}{2a}\pm\sqrt{\frac{b^2-4ac}{4a^2}}$$
$$x=\frac{-b}{2a}\pm\frac{\sqrt{b^2-4ac}}{2a}$$

$$ax^2+bx+c=0 \qquad \boxed{x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}}$$

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

a) $x^2 + 9x + 14 = 0$

$a=1, b=9, c=14$

$$x = \frac{-9 \pm \sqrt{9^2-4(1)(14)}}{2(1)}$$

$$= \frac{-9 \pm \sqrt{81-56}}{2}$$

$$= \frac{-9 \pm \sqrt{25}}{2}$$

$$= \frac{-9 \pm 5}{2} = \frac{-9+5}{2}, \frac{-9-5}{2}$$

$$\boxed{x = -2, -7}$$

b) $2x^2 + 5x - 6 = 0$

$a=2, b=5, c=-6$

$$x = \frac{-5 \pm \sqrt{5^2-4(2)(-6)}}{2(2)}$$

$$x = \frac{-5 \pm \sqrt{25+48}}{4}$$

$$\boxed{x = \frac{-5 \pm \sqrt{73}}{4}}$$

Solve these using the quadratic formula.

a) $2x^2 = 14$

$2x^2 - 14 = 0$

$a = 2,\ b = 0,\ c = -14$

$x = \frac{-0 \pm \sqrt{0^2 - 4(2)(-14)}}{2(2)}$

$= \frac{\pm\sqrt{4 \cdot 4 \cdot 7}}{4}$

$= \frac{\pm 4\sqrt{7}}{4} = \pm\sqrt{7}$

b) $8x^2 - 10x + 3 = 0$

$a = 8,\ b = -10,\ c = 3$

$x = \frac{10 \pm \sqrt{(-10)^2 - 4(8)(3)}}{2(8)}$

$= \frac{10 \pm \sqrt{100 - 96}}{16}$

$= \frac{10 \pm \sqrt{4}}{16} = \frac{10 \pm 2}{16}$

$= \frac{12}{16}, \frac{8}{16}$

$x = \frac{3}{4}, \frac{1}{2}$

Solve using any strategy.

a) $9x^2 + 18x - 135 = 0$

$$9(x^2+2x-15)=0$$

$$\frac{\cancel{9}(x+5)(x-3)}{\cancel{9}}=\frac{0}{9}$$

$$(x+5)(x-3)=0$$

$$x+5=0 \qquad x-3=0$$

$$x=-5 \qquad x=3$$

strategy: factoring

b) $(y+4)^2 - 18 = 0$

$$\sqrt{(y+4)^2}=\pm\sqrt{18}$$

$$y+4=\pm\sqrt{18}$$

$$y=-4\pm\sqrt{18}$$

$$=-4\pm\sqrt{9}\sqrt{2}$$

$$\boxed{y=-4\pm3\sqrt{2}}$$

strategy: square root

c) $5x^2 - 16x = -2$

strategy: quadratic formula

$$5x^2-16x+2=0$$

$$a=5,\ b=-16,\ c=2$$

$$x=\frac{16\pm\sqrt{(-16)^2-4(5)(2)}}{2(5)}$$

$$x=\frac{16\pm\sqrt{256-40}}{10}=\frac{16\pm\sqrt{216}}{10}=\frac{16\pm\sqrt{36}\sqrt{6}}{10}$$

$$x=\frac{16\pm6\sqrt{6}}{10}=\frac{\cancel{2}^{1}(8\pm3\sqrt{6})}{\cancel{10}_{5}}=\boxed{\frac{8\pm3\sqrt{6}}{5}}$$

d) $2y(y-2) = 7$

$2y^2 - 4y = 7$

strategy: quadratic formula

$2y^2 - 4y - 7 = 0$

(not factorable)

$a = 2,\ b = -4,\ c = -7$

$$y = \frac{4 \pm \sqrt{(-4)^2 - 4(2)(-7)}}{2(2)} = \frac{4 \pm \sqrt{16+56}}{4}$$

$$y = \frac{4 \pm \sqrt{72}}{4} = \frac{4 \pm \sqrt{36}\sqrt{2}}{4} = \frac{4 \pm 6\sqrt{2}}{4} = \frac{\cancel{2}^{1}(2 \pm 3\sqrt{2})}{\cancel{4}_{2}}$$

$$= \frac{2 \pm 3\sqrt{2}}{2}$$

e) $x^{2/3} - 9x^{1/3} + 8 = 0$

(quadratic form)

$(x^{1/3})^2 - 9(x^{1/3}) + 8 = 0 \iff$

$y = x^{1/3}$

$y^2 - 9y + 8 = 0$

$(x^{1/3} - 8)(x^{1/3} - 1) = 0$

$x^{1/3} - 8 = 0$ or $x^{1/3} - 1 = 0$

$x^{1/3} = 8$ $\qquad$ $x^{1/3} = 1$

or $(\sqrt[3]{x})^3 = (8)^3$ $\qquad$ $(x^{1/3})^3 = 1^3$

$x = 512$ $\qquad$ $x = 1$