MATH 1010 ~ Intermediate Algebra   Chapter 9: EXPONENTIAL AND
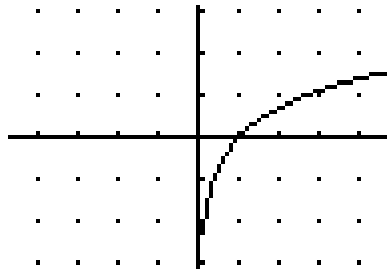   LOGARITHMIC FUNCTIONS

Section 9.3: Logarithmic Functions

Objectives:

   ※   Evaluate logarithmic functions.
   ※   Graph logarithmic functions.

$$\log_2(x) = y \Leftrightarrow 2^y = x$$

$$f(x) = \ln(x)$$

Logarithmic Equations

Defn log
$$y = \log_a x \Longleftrightarrow x = a^y$$

read "log base a of x"

defn division
$$a \div b = c \Longleftrightarrow a = c \cdot b$$

$*$ logarithm fn undoes exponential fn

$y = e^x$

inverse fn

ex $15 \div 5 = ?$ $5 \cdot ? = 15$

Log Properties

$a \neq 0, 1, \ a > 1$

$\log_a 1 = ? \Longleftrightarrow a^? = 1 \quad ? = 0$

① $\boxed{\log_a 1 = 0}$
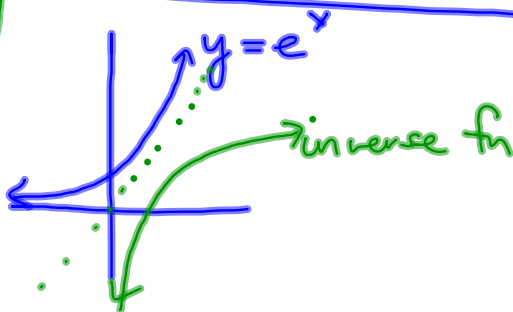
$\log_a a = ? \Longleftrightarrow a^? = a \quad ? = 1$

② $\boxed{\log_a a = 1}$

$\log_a a^x = ? \Longleftrightarrow a^? = a^x \quad ? = x$

③ $\boxed{\log_a a^x = x}$

Defn $\log_\heartsuit \square = \bigstar \Longleftrightarrow \heartsuit^{\bigstar} = \square$

ex $\log_5 25 = ? \Longleftrightarrow 5^? = 25$

$5 = \heartsuit \quad 25 = \square \quad ? = \bigstar$

① EXAMPLE

Evaluate these expressions.

a) $\log_6 1 = ? \ = 0$

$\Longleftrightarrow \quad 6^? = 1$

b) $\log_{10}\left(\dfrac{1}{100}\right) = ? = -2$

$10^? = \dfrac{1}{100} = \dfrac{1}{10^2} = 10^{-2}$

c) $\log_4(-1) = ?$

$?$ DNE

$4^? = -1$

d) $\log_5(0) = ?$

$?$ undefined

$5^? = 0$

e) $\log_{144} 12 = \dfrac{1}{2}$

$144^? = 12$

$(12^2)^? = 12$

$144^? = 12$

$\sqrt{144} = 12$

$? = \frac{1}{2}$

$12^{2?} = 12^1$

$2(?) = 1$

$? = \frac{1}{2}$

f) $\log_2(256) = 8$

$?$

$2^? = 256$

$2^8 = 256$

② EXAMPLE

Rewrite in the other format.

Defn

$$\log_{\heartsuit} \square = \star$$
$$(\Leftrightarrow) \quad \heartsuit^{\star} = \square$$

a) $\log_{32} 4 = \dfrac{2}{5}$

$32 = \heartsuit \quad 4 = \square \quad \dfrac{2}{5} = \star$

$32^{2/5} = 4$

b) $\log_3 \dfrac{1}{27} = -3$

$3^{-3} = \dfrac{1}{27}$

c) $6^{-3} = \dfrac{1}{216}$

$\heartsuit = 6 \quad \star = -3 \quad \square = \dfrac{1}{216}$
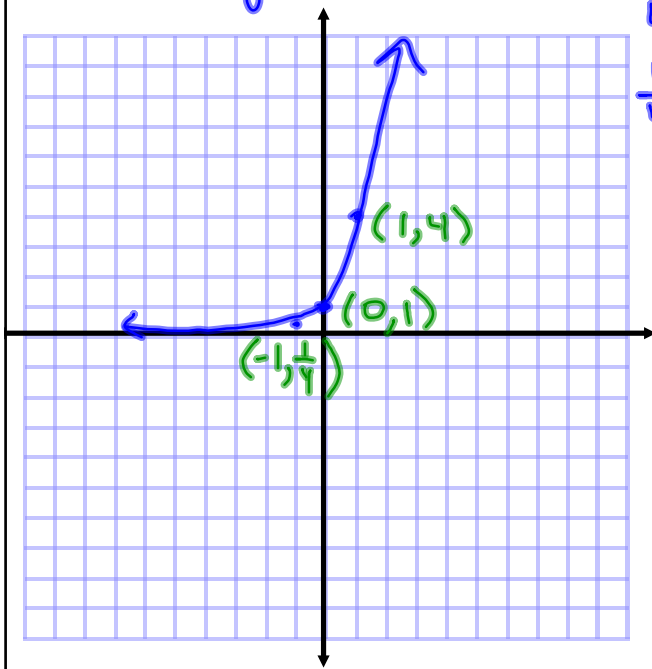
$\log_6 \left(\dfrac{1}{216}\right) = -3$

d) $4^1 = 4$

$\log_4 4 = 1$

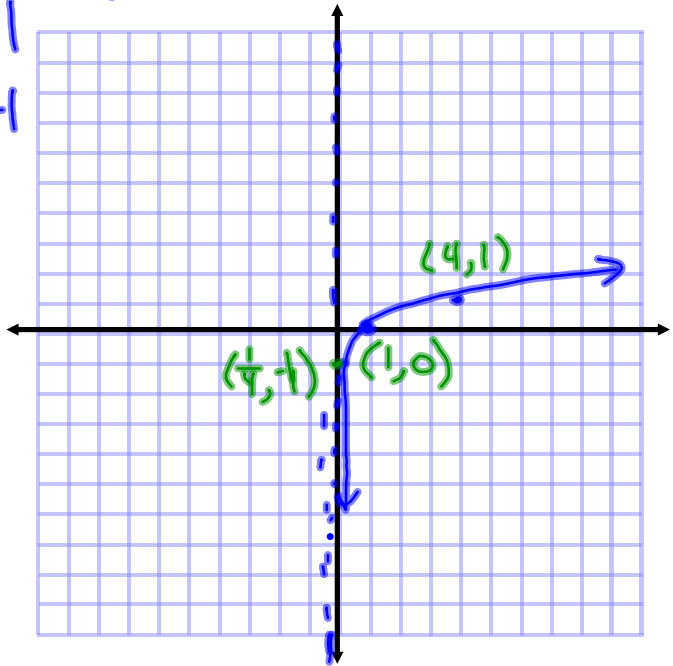Graphs and Vertical Asymptotes $\log_4 x = y \iff 4^y = x$

$y = 4^x$

HA: $y = 0$

$y = \log_4 x$

VA: $x = 0$

x	y
1	0
4	1
$\frac{1}{4}$	-1

$y = \log_4 1$

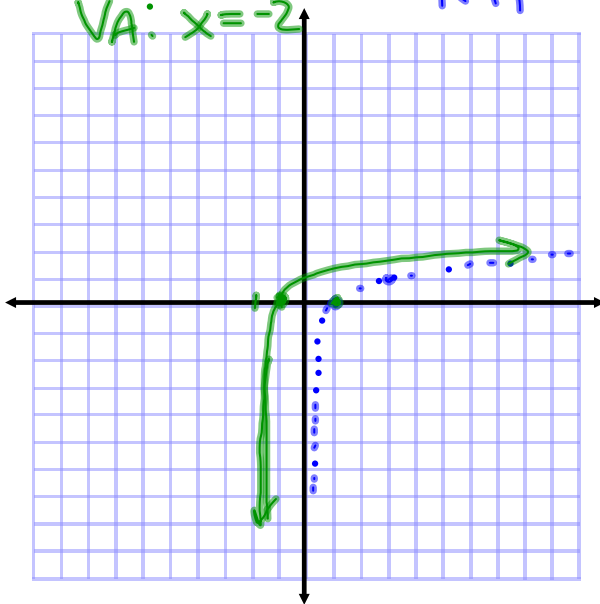$(1,4)$
$(0,1)$
$(-1, \frac{1}{4})$

$(4,1)$
$(\frac{1}{4},-1)$ $(1,0)$
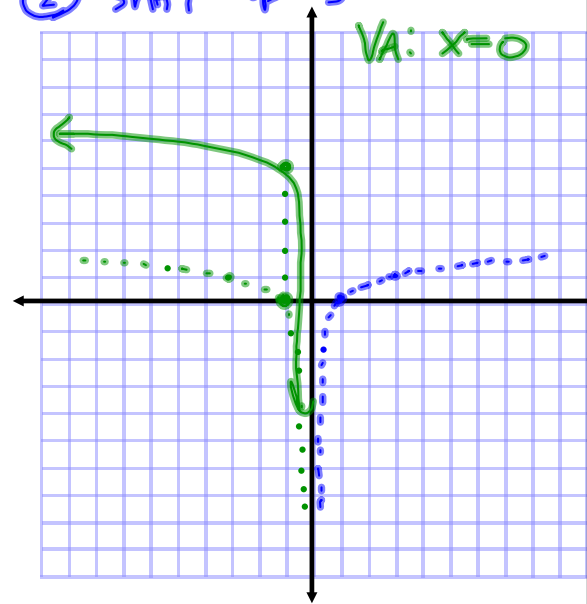
$f(x) = \log_a x$

domain: $x > 0$

a) $y = \log_3(x + 2)$

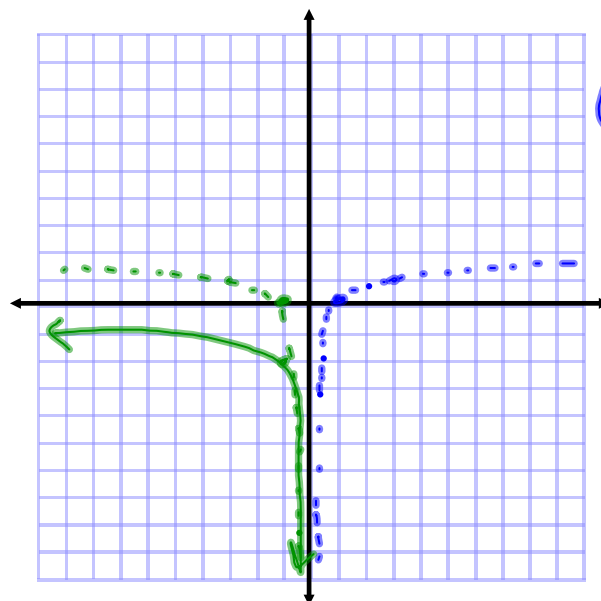base: $y = \log_3 x$ shift 2 left

VA: $x = -2$

b) $y = \log_3(-x) + 5$

① reflection: horizontal
② shift up 5

VA: $x = 0$

c) $y = \log_3(-x) - 2$

① reflection: horizontal
② shift down 2

VA: $x = 0$