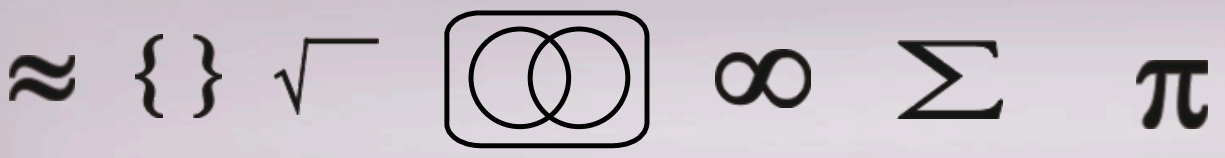

FRM

ARM

Math 1030 #10a

Loans, Credit Cards and Mortgages

Loans

Installment Loan

Payday Loan

Loans, credit cards and mortgages are ways you borrow money. When you borrow money, the bank is basically making a lump sum investment and getting a periodic return.

Principal is the amount of money owed at any particular time.

Interest is charged on the principal.

To pay off a loan, you should pay the interest and also some part of the principal.

An installment loan (amortized loan) is a loan payed off with equal regular payments.

Loan Payment Formula (Installment loans)

$$PMT = \frac{P \cdot \left(\frac{APR}{n} \right)}{\left[1 - \left(1 + \frac{APR}{n} \right)^{(-nY)} \right]}$$

PMT = regular payment amount

P = starting principal

APR = annual percentage rate

n = number of payments per year

Y = the term of the loan (years)

EX 1: You have a student loan of $40,000 with an APR of 6%. Compare monthly payment amounts and total amount paid for these options.

$P = 40{,}000$

$n = 12$

$APR = 0.06$

$$PMT = \frac{P \cdot \left(\frac{APR}{n}\right)}{\left[1 - \left(1 + \frac{APR}{n}\right)^{(-nY)}\right]}$$

15 years

$Y = 15$

$$PMT = \frac{40000\left(\frac{0.06}{12}\right)}{1 - \left(1 + \frac{0.06}{12}\right)^{-12(15)}}$$

$$= \frac{40000(0.005)}{1 - 1.005^{-180}}$$

$\simeq \$337.54$

total paid (over the 15 yrs) $= 337.54(12)(15)$

$\simeq \$60{,}757.20$

25 years

$Y = 25$

$$PMT = \frac{40000\,(0.005)}{1 - 1.005^{-12(25)}}$$

$\simeq \$257.72$

total paid:

$257.72(12)(25)$

$\simeq \$77{,}316.00$

EX 2: For the loan in example 1, make a table showing the amounts of each monthly payment that goes to the principal and to the interest for the first three months. ($40,000 at 6% for 15 yrs)

$\frac{0.06}{12} = 0.005$

	Loan balance	interest	payment	principal paid
1	40000	$40000(0.005) = 200$	337.54	$337.54 - 200 = 137.54$
2	$40000 - 137.54 = 39862.46$	$39862.46\,(0.005) = 199.31$	337.54	$337.54 - 199.31 = 138.23$
3	$39862.46 - 138.23 = 39724.23$	$39724.23\,(0.005) = 198.62$	337.54	$337.54 - 198.62 = 138.92$

EX 3: You borrow $4000 to buy a used car. You can afford monthly payments of $150. Which of these meets your needs?

$$PMT = \frac{P \cdot \left(\frac{APR}{n}\right)}{\left[1 - \left(1 + \frac{APR}{n}\right)^{(-nY)}\right]}$$

(1) 2 years at 8% APR

$$PMT = \frac{4000\left(\frac{0.08}{12}\right)}{\left(1-\left(1+\frac{0.08}{12}\right)^{-12(2)}\right)} \approx \$180.90$$

(2) 3 years at 9% APR

$$PMT = \frac{4000\left(\frac{0.09}{12}\right)}{\left(1-\left(1+\frac{0.09}{12}\right)^{-12(3)}\right)} \approx \$127.19$$

(3) 4 years at 10% APR

$$PMT = \frac{4000\left(\frac{0.10}{12}\right)}{\left(1-\left(1+\frac{0.10}{12}\right)^{-12(4)}\right)} \approx \$101.45$$

EX 4: A payday loan company charges $150 to borrow $1000 for 2 weeks. What is the APR?

$150 is 15% of $1000 (only for 2 weeks)

$$\frac{15\%}{2 \text{ weeks}} \cdot \frac{52 \text{ wks}}{1 \text{ yr}} = 15\% \frac{(26)}{\text{yr}} = 390\% \text{ (per year)}$$

$$\Rightarrow APR = 390\%$$