Math 1030 #12b

Doubling Time and Half-Life

Half-Life

Exponential Decay

Exponential Growth

Half-life

Doubling

Half-life is the time it takes for half of a population to vanish if it decreases by the same percent each time period. exponential decay scenario

EX 1: If a city of 1000 people is decreasing by 10% each year, when will there be half as many people?

Year	Number of people
0	1000
1	$0.9(1000) = 900$
2	$0.9^2(1000) = 810$
3	$0.9(0.9^2)(1000) = 0.9^3(1000) = 729$
4	$0.9^4(1000) \simeq 656$
5	$0.9^5(1000) \simeq 595$
6	$0.9^6(1000) \simeq 531$
7	$0.9^7(1000) \simeq 478$

$\Rightarrow$ half-life is about 7 yrs

After a time, t, an exponentially decaying quantity with a half life of T_{half} decreases in size by a factor of $(1/2)^{t/T}$. The new value is related to the initial value by $\boxed{\textit{new value} = \textit{initial value} \times (1/2)^{t/T}}$.

decay (shrink) factor $= \left(\frac{1}{2}\right)^{t/T}$

<u>Approximate Half-life</u>

For a quantity decaying exponentially at a rate of P% per time period

$$\boxed{T_{half} \approx \frac{70}{P}}$$

This works best for small rates and breaks down for rates over about 15%.

EX 2: Radioactive carbon-14 has a half-life of about 5700 years. It collects in organisms only while they are alive. Once they are dead, it only decays. What fraction of carbon-14 in an animal bone still remains 800 years after the animal has died?

$T_{half} = 5700$ yrs $t = 800$ yrs

decreases (shrinks) by factor

of $\left(\frac{1}{2}\right)^{\frac{800}{5700}} \simeq 0.9073$

$\Rightarrow$ about 91% of carbon-14 still remains after 800 yrs

EX 3: A clean-up project is reducing the concentration of a pollutant in the water supply, with a 3.5% decrease per week.

time period

a) What is the approximate half-life of the concentration of the pollutant?

$$T \approx \frac{70}{3.5} = 20 \text{ weeks}$$

b) What fraction of the original will remain after one year?

1 yr = 52 weeks, $t = 52$ wks, $T_h = 20$ wks

decay (shrink) factor $= \left(\frac{1}{2}\right)^{t/T}$

$= \left(\frac{1}{2}\right)^{52/20} \simeq 0.165$

$\Rightarrow$ about 16.5% of original pollutants remain after 1 yr

Exact half-life formula:

$$\boxed{T_{half} = -\frac{log_{10}(2)}{log_{10}(1+r)}}$$ where r is a decimal and negative.

Note: The units of time for r and T must be the same (per month, year, etc.)

EX 4: Suppose the Russian ruble is falling in value against the dollar at 11% per year. $r = -0.11$, $P = 11$

a) Approximately how long will it take the ruble to lose half its value?

$T_h \approx \frac{70}{11} \simeq 6.36$ yrs

b) Exactly how long will it take the ruble to lose half its value?

$$T_h = \frac{-\log 2}{\log(1 + -0.11)} \simeq \frac{-\log 2}{\log(0.89)} \simeq 5.948$$