Math 1030 #15C

More about Carrying Capacity and Logistic Growth

The *carrying capacity* of a biological species in an environment is the maximum population size of the species that the environment can sustain indefinitely, given the food, habitat, water, and other necessities available in the environment.

The rate of population growth decreases as the population approaches the carrying capacity. If the population equals carrying capacity, the growth rate is 0.

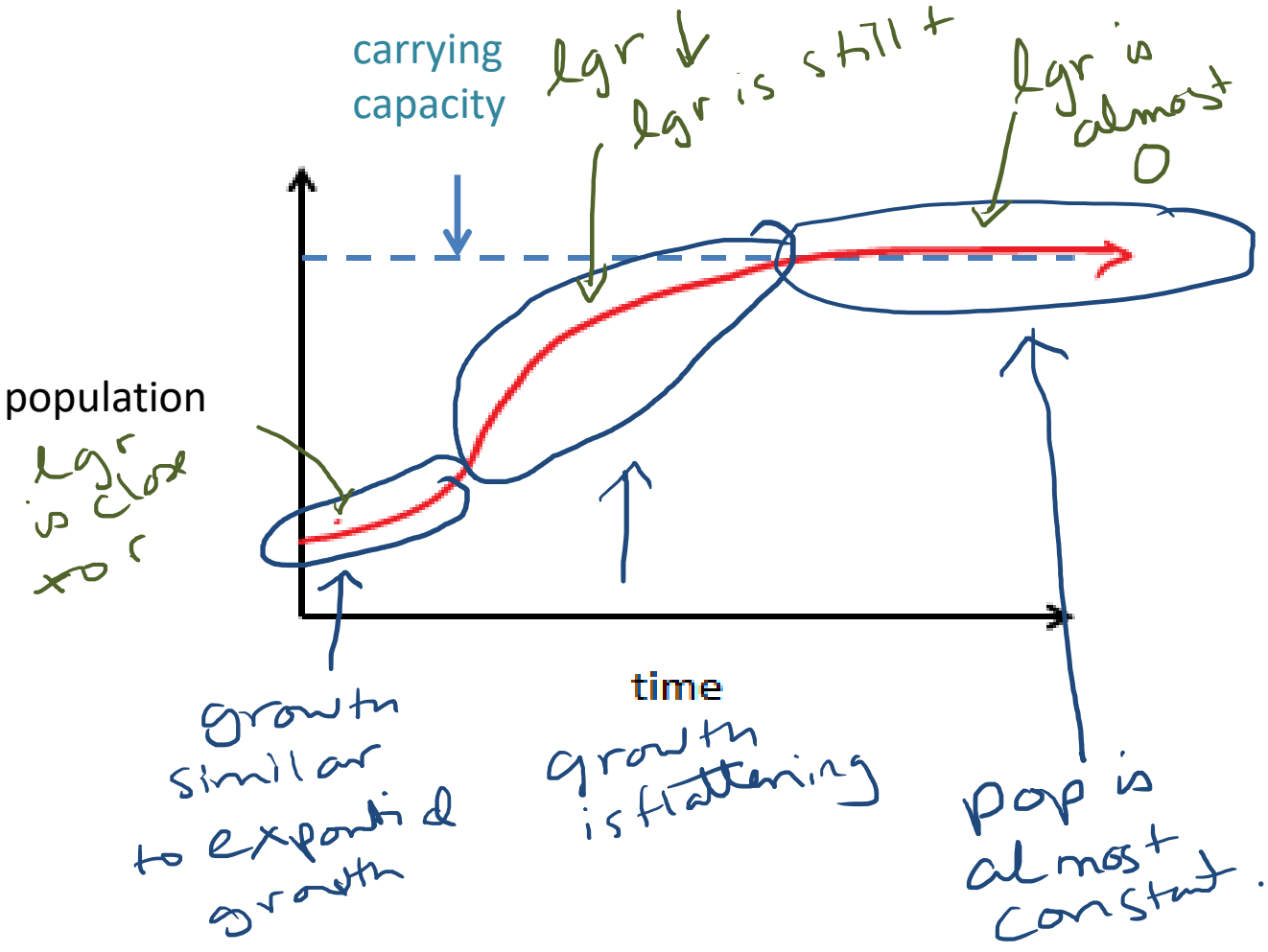
The Logistic Growth Model is based on the assumption that the growth rate decreases smoothly and becomes 0 when the carrying capacity is reached.

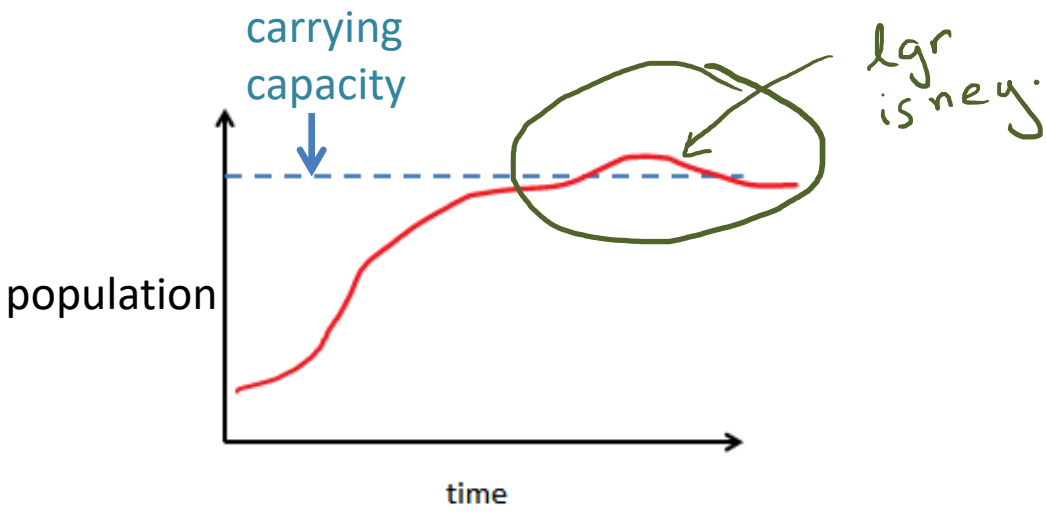
$$lgr = r \cdot \left(1 - \frac{population}{carrying\ capacity}\right)$$

lgr = logistic growth rate

r = initial/base/overall growth rate

A graph of what logistic growth looks like:





The *carrying capacity of Earth* is number of people that Earth can support for long period of time.

Ex 1: The earth's population appears to have been growing logistically since 1960. If the base growth rate was 2.80%, the current population is 7.74 billion and the current growth rate is 1.08%, what would the projected carrying capacity be?

$$lgr = r \cdot \left(1 - \frac{population}{\boldsymbol{carrying\ capacity}}\right)$$

$r = 0.028$

$pop = 7.74$ bil

$lgr = 0.0108$

$cc = ?$

$$0.0108 = 0.028\left(1 - \frac{7.74}{cc}\right)$$

$$\frac{0.0108}{0.028} = 1 - \frac{7.74}{cc}$$

$$\frac{0.0108}{0.028} - 1 = -\frac{7.74}{cc}$$

$$cc\left(\frac{0.0108}{0.028} - 1\right) = -7.74$$

$$cc = \frac{-7.74}{\left(\frac{0.0108}{0.028} - 1\right)}$$

$cc = 12.6$ bil

Ex 2 Suppose the population of bacteria in a petri dish is growing according to a logistic growth model and that it has a base growth rate of 15%. The carrying capacity of the petri dish is 90 billion bacteria.

a. What is the growth rate when there are 50 billion bacteria?

$r = 0.15$
pop = 50 bil
CC = 90 bil
lgr = ?

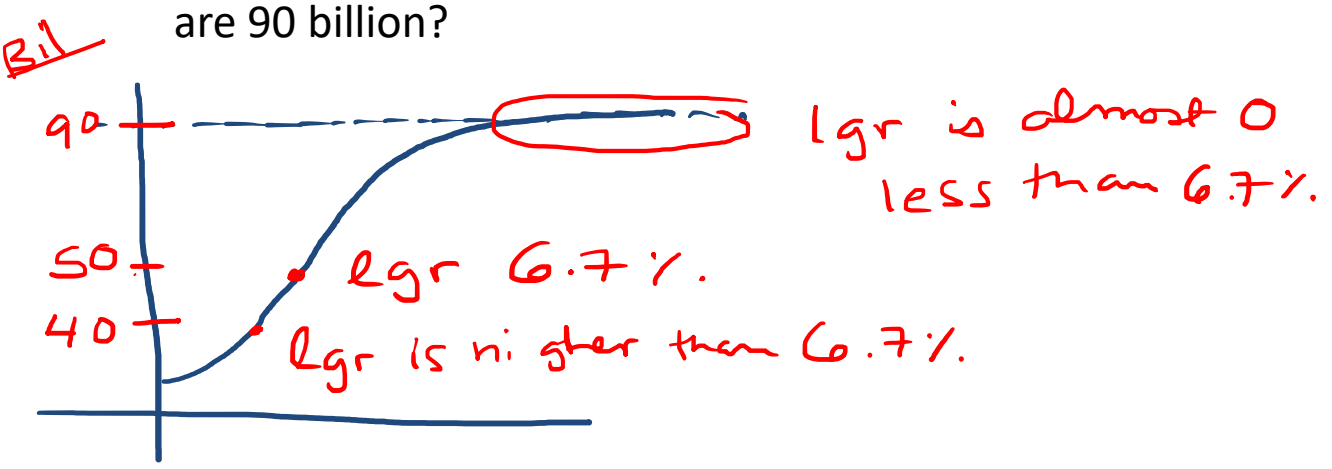
$$\textbf{lgr} = r \cdot \left(1 - \frac{population}{carrying\ capacity}\right)$$

$$lgr = 0.15\left(1 - \frac{50\ bil}{90\ bil}\right)$$

$$\approx 0.0666\ldots$$

6.7%.

b. Is the growth rate higher or lower than the answer for part a. when there are 40 billion bacteria? When there are 90 billion?



c. Suppose the population is 95 billion. Find and interpret the growth rate.

lgr

$lgr = 0.15\left(1 - \frac{95\ bil}{90\ bil}\right)$

$= -0.008333\ldots$

$lgr \approx -0.8\%$

r = 0.15
pop = 95 bil
CC = 90 bil
lgr = ?

lgr is negative because we exceeded carrying capacity.