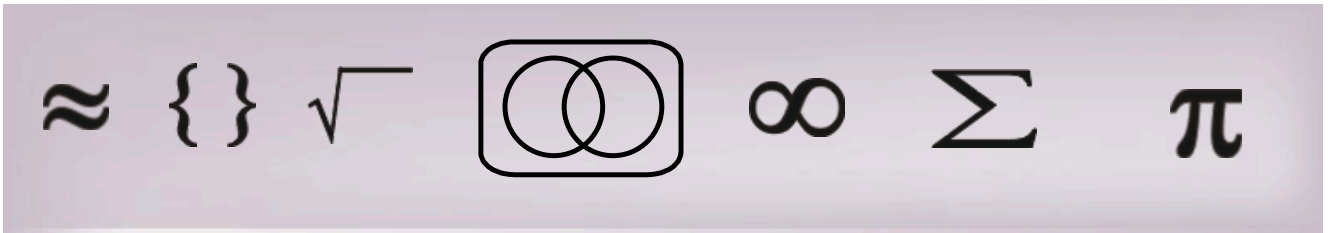

exponential growth

exponential decay

Math 1030 #16a

Exponential Modeling

growth rate

rate of decay

Exponential Functions

An Exponential Function is one in which the *relative growth rate* is constant.

Consider this example:

EX 1: One of the top five fastest growing small towns in the USA is Heber, Utah which had a population of 20,000 in 2007. The population grows by 15% each year.

Population after *1* year $= 20{,}000 \times 1.15 =$ 23,000

Population after *2* years $= (20{,}000 \times 1.15) \times (1.15)$

$= 20000(1.15)^2 = 26450$

Population after *3* years $= 20{,}000 \times 1.15 \times 1.15 \times 1.15$

$= 20000(1.15)^3$

$\simeq 30418$

Population after t years = initial population $\times 1.15^t$

$= 20000(1.15)^t$

What is the population of Heber in 2014?

$t = 7$ yrs, $\quad pop = 20000(1.15^7) \simeq 53{,}200$

Can we write a general equation for this function?

(annual) growth rate of r, initial population of P_0

t = time (yrs)

$$P(t) = P = P_0(1+r)^t$$

(note: r can be positive (growth) or negative (decay/shrink))

Exponential functions grow (or decay) by the same relative amount per unit of time.

For any quantity Q growing exponentially with a fractional growth rate r,

$$Q = Q_0(1+r)^t$$

(r is growth rate per time period that matches how t is measured)

where

Q = value of the exponentially growing quantity at time t. (it's a function of t)

Q_0 = initial value of the quantity (at $t=0$)

r = fractional growth rate (which may be positive or negative)

t = time

EX 2: The number of restaurants in a city is growing according to this equation, beginning in the year 2010. $Q = 500(1.03)^t$

(note: $t=0$ in yr 2010)

Q_0 $1+r$

a) What is the rate of growth?

because we know $Q = Q_0(1+r)^t$

$\Rightarrow r = 0.03 = 3\%$

b) How many restaurants were there in the year 2010?

$\Rightarrow Q_0 = 500$

c) How many restaurants will there be in 2020?

in yr 2020, $t = 10$ yrs, $Q = ?$

$Q = 500(1.03)^t$

$Q = 500(1.03)^{10} \simeq 672$ restaurants in 2020

EX 3: The population of Cook Islands has been decreasing. The rate of decrease is 3% each year. In 2012 there were 11,000 people on the Islands. $r=-0.03$, time period = 1 yr, $Q_0=11000$ people

a) Write an equation for the decline in population.

$t=0$ in 2012

$Q=Q_0(1+r)^t$

$Q=11000(1+-0.03)^t$

$\boxed{Q=11000(0.97)^t}$

b) At this rate, what will the population be in 2025?

$t=2025-2012=13$ yrs., $Q=?$

$Q=11000(0.97)^{13}\simeq 7404$ people