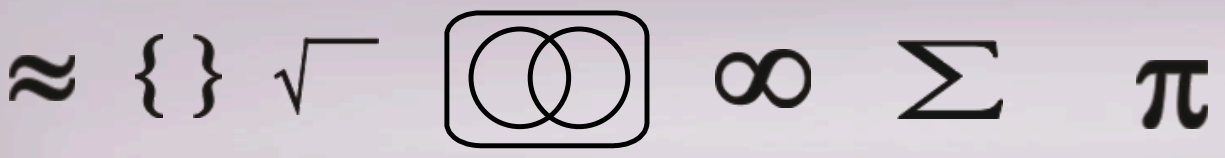

Future Value

Total Return

Math 1030 #9a

Savings Plans and Investments

Investments

Savings Plan Formulas

Annual Return

$A = P(1+APR)^Y$

A = Account balance after Y years
P = Principal amount invested
APR = annual percentage rate (as a decimal)
Y = number of years

(Compound int. formula; compounding annually)

Savings Plan Formula (regular payments)

Suppose you invest $1000 at the end of each year for 5 years in an account that pays 10% interest compounded annually. What is the value after 5 years (future value)?

Year	amount generated	
1	$1000(1+0.1)^4$	$1464.10
2	$1000(1+0.1)^3$	$1331.00
3	$1000(1+0.1)^2$	$1210.00
4	$1000(1+0.1)^1$	$1100.00
5	$1000(1+0.1)^0$	$1000.00
Total		$6105.10

($1000 earns 4 yrs of interest)

note: $(1.1)^0 = 1$

$$A = PMT \cdot \frac{(1+\frac{APR}{n})^{nY} - 1}{\frac{APR}{n}}$$

A = balance after Y years
APR = annual interest rate
n = number of payment periods per year
Y = number of years
PMT = regular payment amount

Savings plan formula (where you deposit/invest same amt of $ every period and want to know future value)

(also called future value of an ordinary annuity)

$$A = PMT \cdot \frac{\left(1 + \frac{APR}{n}\right)^{nY} - 1}{\frac{APR}{n}}$$

A = balance after Y years
APR = annual interest rate
n = number of payment periods per year
Y = number of years
PMT = regular payment amount

EX 1: Find the savings plan balance after 5 years with an APR of 2.5% with monthly payments of \$100.

(hidden assumption: compounding occurs monthly, i.e. payment periods match compounding period)

$Y=5,\ APR=0.025,\ PMT=100,\ n=12$

$$A = 100\frac{\left[\left(1+\frac{0.025}{12}\right)^{12(5)} - 1\right]}{\frac{0.025}{12}}$$

$$= 100\frac{\left[(1.00208\overline{3})^{60} - 1\right]}{0.00208\overline{3}}$$

$$\simeq \boxed{\$6,384.05}$$

Note: we put in $100(12)(5) = \$6000$.

EX 2: At age 28 you begin saving $50 at the end of each month in an account with an APR of 4%. How much will the balance be when you retire at age 65? How does this compare to the amount invested?

$$A = PMT \cdot \frac{(1+\frac{APR}{n})^{nY} - 1}{\frac{APR}{n}}$$

PMT = \$50, n = 12, APR = 0.04

Y = 65 − 28 = 37

$$A = 50\left[\frac{\left(1+\frac{0.04}{12}\right)^{(12(37))} - 1}{\frac{0.04}{12}}\right]$$

$\simeq \$50{,}732.21$

Compare w/ total amt deposited: $50(12)(37) = \$22{,}200.00$

EX 3: At age 23 when you graduate, you start saving for retirement. Your investment plan pays an APR of 4.5%. You want to have \$5 million when you retire in 45 years. How much should you deposit monthly?

$$A = PMT \cdot \frac{(1+\frac{APR}{n})^{nY} - 1}{\frac{APR}{n}}$$

APR = 0.045, A = \$5,000,000

Y = 45, n = 12

$$5000000 = \frac{PMT\left[\left(1+\frac{0.045}{12}\right)^{(12(45))} - 1\right]}{\frac{0.045}{12}}$$

$$5{,}000{,}000 = \frac{PMT\left[1.00375^{540} - 1\right]}{0.00375}$$

$5{,}000{,}000 \simeq (1745.992368)PMT$

$\$2863.70 \simeq PMT$

$2863.70(12)(45) \simeq \$1{,}546{,}398$