REVIEW

Properties
of
Logarithms

Remember this:

1. $y = b^x \Leftrightarrow x = \log_b y$ $\quad b > 0,\ b \neq 1$
2. $x = \log_{10} y \Leftrightarrow x = \log y$
3. $x = \log_e y \Leftrightarrow x = \ln y$
4. $\log 10^x = x$
5. $\ln e^x = x$

Try these:

a) *log (1000)* = 3 $\quad 10^{?} = 1000$

b) *ln* (e^5) = 5 $\quad e^{?} = e^{5}$

c) *log (1)* = 0 $\quad 10^{?} = 1$

d) *ln (1)* = 0 $\quad e^{?} = 1$

Properties of Logarithms

1) $\log(uv) = \log u + \log v$

2) $\log\left(\frac{u}{v}\right) = \log u - \log v$

3) $\log u^n = n \log u$

WARNING:

(A) $\log(x+y) \neq \log x + \log y$

(B) $(\log u)^n \neq \log u^n$

Simplify these:

a) $\log(25)+\log(40) \overset{①}{=} \log(25 \cdot 40)$
$= \log(1000) = 3$

b) $\ln 2+\ln 6 \overset{①}{=} \ln(2 \cdot 6)$
$= \ln(12)$

c) $\log 450 - \log 9 \overset{②}{=} \log\left(\frac{450}{9}\right)$
$= \log(50)$

d) $\ln(10)^6 \overset{③}{=} 6 \ln 10$

e) $\ln(e^2 \cdot e^4) = \ln(e^6)$
$\overset{②}{=} 6 \ln e = 6(1) = 6$

f) $\log(.0006) = \log(6 \cdot 10^{-4})$
$\overset{①}{=} \log 6 + \log 10^{-4}$
$\overset{③}{=} \log 6 - 4 \log 10$
$= \log 6 - 4(1) = \log 6 - 4$

note: $= \log(6) - 4$
$= -4 + \log 6$
$\neq \log(6-4)$

Expand these expressions using the properties of logarithms.

a) $\log(5x) =$ ① $\log 5 + \log x$

b) $\ln(x(x-1)^2) =$ ① $\ln x + \ln(x-1)^2$

③ $= \ln x + 2\ln(x-1)$

c) $\log\sqrt{xy} =$ $\log(xy)^{1/2}$ ③ $= \frac{1}{2}\log(xy)$

① $= \frac{1}{2}(\log x + \log y)$

d) $\ln(e^{-2}) =$ ③ $-2\ln e$

$= -2(1) = -2$

Condense these expressions using properties of logarithms.

a) $\log(2x+3y) =$ tricky $\log_7(2x+3y)$

b) $\ln x^5 + \ln(x+1) =$ ① $\ln\left(x^5(x+1)\right)$

c) $3\log x - 2\log y =$ ③ $\log x^3 - \log y^2$ ② $= \log\left(\frac{x^3}{y^2}\right)$

d) $3[\ln x - 2\ln y] =$ ③ $3(\ln x - \ln y^2)$

② $= 3\left(\ln\left(\frac{x}{y^2}\right)\right)$

③ $= \ln\left(\frac{x}{y^2}\right)^3$ or $= \ln\left(\frac{x^3}{y^6}\right)$