2.3 Polynomials and Synthetic Division

- Use long division to divide a polynomial by a polynomial
- Use synthetic division to divide polynomials by a binomial
- Use the Remainder Theorem and Factor Theorem

Review of long division algorithm:

$$\frac{7852}{34}$$

$$\begin{array}{r} 230 \quad R\,32 \\ 34\overline{)7852} \\ \underline{68} \\ 105 \\ \underline{102} \\ 32 \end{array}$$

Polynomial division:

$$\frac{4x^3 - 8x^2 + x - 2}{2x-1}$$

$$\begin{array}{r} 2x^2 - 3x - 1 \quad R\,-3 \\ 2x-1\overline{)4x^3 - 8x^2 + x - 2} \\ \underline{4x^3 - 2x^2} \\ -6x^2 + x \\ \underline{-6x^2 + 3x} \\ -2x - 2 \\ \underline{-2x + 1} \\ -3 \end{array}$$

If the remainder is zero, what does that mean?

Synthetic division - a shortcut

$$\frac{P(x)}{x-r} \qquad \frac{3x^3+5x^2-3x+27}{x+3}$$

$$\begin{array}{r} 3x^2-4x+9 \\ x+3\ \overline{)\,3x^3+5x^2-3x+27x^0} \\ \underline{3x^3+9x^2} \\ -4x^2-3x \\ \underline{-4x^2-12x} \\ 9x+27 \\ \underline{9x+27} \\ \end{array}$$

$3x^3+5x^2-3x+27=(x+3)(3x^2-4x+9)$

$P(-3)= 3(-27)+5\cdot 9-3(-3)+27$

$-81+45+9+27$

$=0$

Synthetic Division

coeff.

$$\begin{array}{r|rrrr} -3 & 3 & 5 & -3 & 27 \\ & & -9 & 12 & -27 \\ \hline & 3 & -4 & 9 & \end{array}$$

$\frac{x^3+1}{x+1}$

$$\begin{array}{r|rrrr} -1 & 1x^3 & 0x^2 & 0x & 1 \\ & & -1 & 1 & -1 \\ \hline & 1x^2 & -1x & 1 & \end{array}$$

$(x+1)(x^2-x+1)$

$$\frac{P(x)}{(x-r)}$$

$$P(r) = 0$$

If the remainder is zero, then x-r is a factor of P(x) and P(r) = 0.

$$P(x) = 3x^3+4x^2+8$$

P(1) $= 3+4+8$
15

Divide by (x-1)

$$\begin{array}{r|rrrr} 1 & 3 & 4 & 0 & 8 \\ & & 3 & 7 & 7 \\ \hline & 3 & 7 & 7 & 15 \end{array}$$

$3x^2+7x+7 \quad R15$

P(-4) $= 3(-64)+4(16)+8$
$-192+64+8$
$+72$
-120

Divide by (x+4)

$$\begin{array}{r|rrrr} -4 & 3 & 4 & 0 & 8 \\ & & -12 & 32 & -128 \\ \hline & 3 & -8 & 32 & -120 \end{array}$$

P(-2) =
$3(-8)+4(4)+8$
$-24+16+8$
= ☺

Divide by x+2

$$\begin{array}{r|rrrr} -2 & 3 & 4 & 0 & 8 \\ & & -6 & 4 & -8 \\ \hline & 3 & -2 & 4 & ☺ \end{array}$$