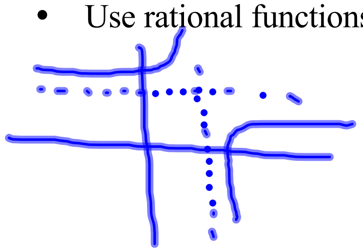
2.6 RATIONAL FUNCTIONS

In this section you will learn how to:

- Find the domain of rational functions
- Find horizontal, vertical and slant asymptotes of rational functions
- Analyze and sketch the graph of a rational function
- Use rational functions to model and solve real-life problems



A rational function is $Q(x) = \dfrac{N(x)}{D(x)}$

$Q(x)\ \dfrac{3x-2}{x^2+5}$ $P(x) = \dfrac{x^2+x}{5}$ Not

where N(x) is a polynomial function of any degree and D(x) must be a polynomial of degree 1 or greater.

$F(x) = \dfrac{3}{x-1}$

The Numerator determines the roots and the Denominator determines the vertical asymptotes.

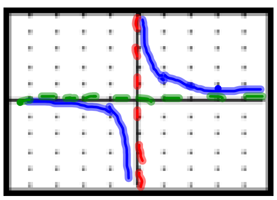
Vertical Asymptotes are caused by zero values in the denominator.

A look at $y = \frac{1}{x}$ and some transformations

vertical asymptote $x = 0$

$y = \frac{1}{x}$

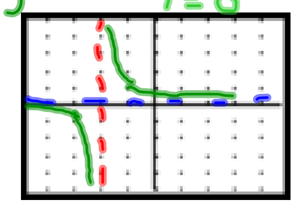
Horizontal asymp. $y = 0$



$x = -2$

$y = \frac{1}{x+2}$

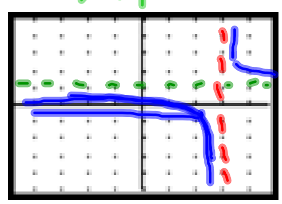
$y = 0$



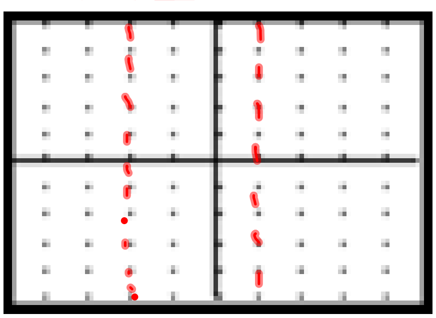
$x = 3$

$y = \frac{1}{x-3} + 1$

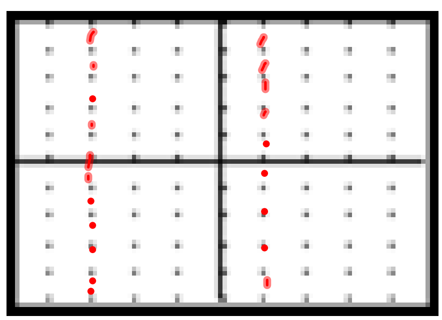
$y = 1$



$y = \frac{2x - 3}{(x-1)(x+2)}$



$y = \frac{2x(x+2)}{(x-1)(x+3)}$



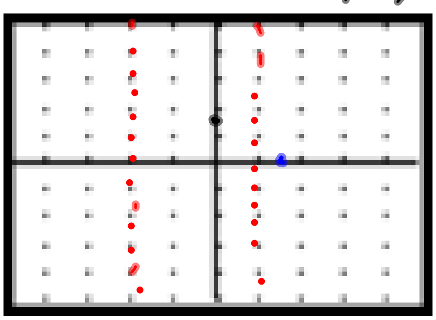
Vertical asymp.
$D(x) = 0$
Set. Denom $= 0$
Solve

The numerator tell us the roots (x-intercepts) of the function.
To find the y-intercept, let x=0.

Roots (x-intercepts) $N(x)=0$

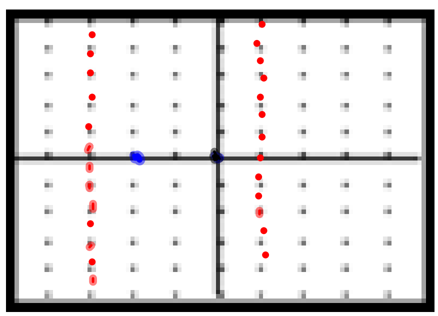
$$y = \frac{2x - 3}{(x-1)(x+2)}$$

$$x=0 \Rightarrow y = \frac{-3}{(-1)(2)} = \frac{3}{2}$$



$$y = \frac{2x(x+2)}{(x-1)(x+3)}$$

$$\frac{0(2)}{(-1)(3)} = \frac{0}{-3} = 0$$



End behavior is determined by the quotient of the leading terms.

$y = \frac{2x}{x^2} = \frac{2}{x}$

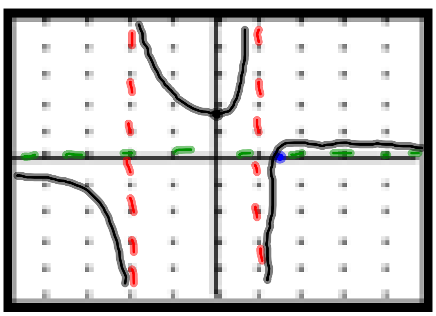
$y = \frac{2x - 3}{(x-1)(x+2)}$

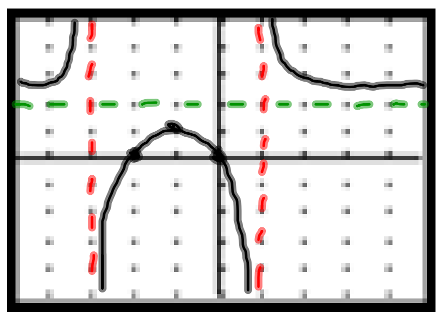
EB: H.A $y = 0$

$y = \frac{2x^2}{x^2} = 2$

$y = \frac{2x(x+2)}{(x-1)(x+3)}$

EB $y = 2$





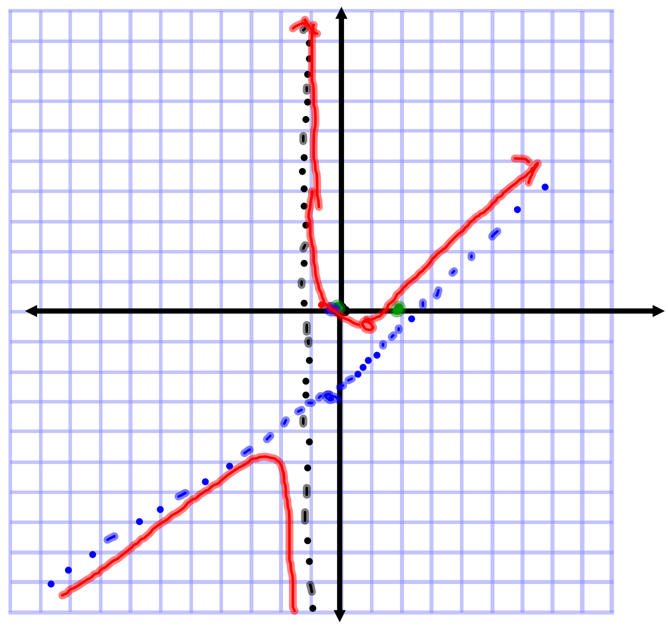
How do we know what the function looks like? We need to make a sign line:

What happens if there is a common factor in the numerator and the denominator?

$$y = \frac{x^3 - 3x^2 + 2x}{x^2 - 1}$$

$$y = \frac{x(x^2-3x+2)}{(x-1)(x+1)}$$

$$y = \frac{x(x-2)\cancel{(x-1)}}{\cancel{(x-1)}(x+1)}$$



hole in the function: hole:

at $x=1$ $\frac{1(-1)}{2} = -\frac{1}{2}$ $(1, -\frac{1}{2})$

Roots: $x=0$ $x-2=0$

$(0,0)$ $(2,0)$

y-int $x=0$ $\frac{0(-2)}{1} = 0$

V. Asymptotes Denom $=0$

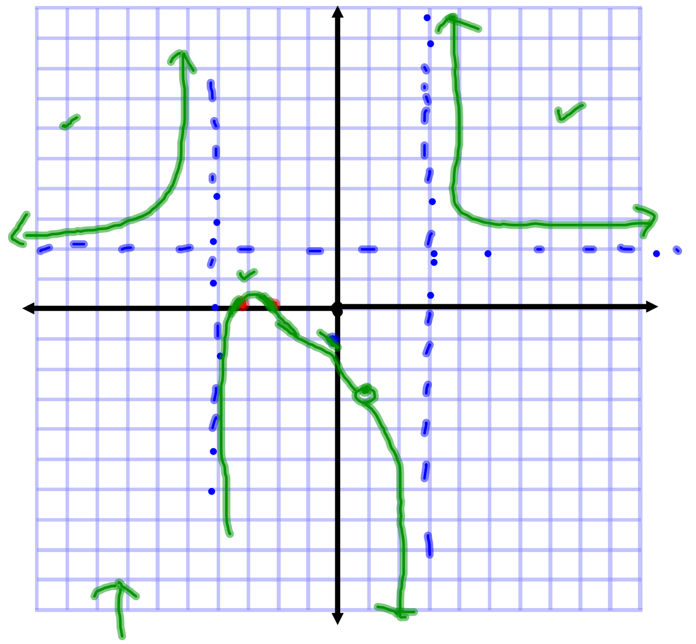
$x+1=0$ $x=-1$

End Behavior $y = \frac{x^2 - 2x}{x+1}$

$$\begin{array}{r} x-3 \\ x+1 \overline{)\, x^2 - 2x} \\ x^2 + x \\ \hline 0 - 3x \\ -3x - 3 \end{array}$$

asymptote $y = x-3$

$$y = \frac{(x+2)\cancel{(x-1)}(2x+6)}{\cancel{(x-1)}(x+4)(x-3)}$$



IN SUMMARY:

Factor numerator and denominator

Reduce any common factors and note the hole(s) in the function.

$$\frac{3 \cdot 8}{5 \cdot (-2)} = \frac{24}{-10} = -2.4 \quad \text{hole } (1, -2.4)$$

Determine x and y intercepts.

$$y = \frac{(x+2)(2x+6)}{(x+4)(x-3)}$$

at $x = -2$, $x = -3$

$(-2, 0)$ $(-3, 0)$

$$y = \frac{2 \cdot 6}{4(-3)} = -1 \quad (0, -1)$$

Determine end behavior. h.A.

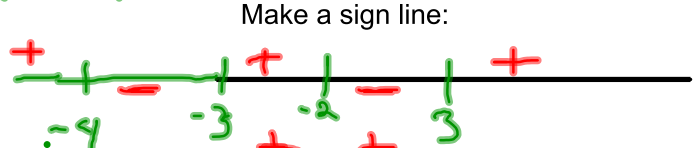
$$y = \frac{2x^2 + \ldots}{x^2 + \ldots} = 2$$

$y = 2$

Determine vertical asymptotes. (Denom)

$x = -4$ $x = 3$

Make a sign line:



$$\frac{(x+2)(2x+6)}{(x+4)(x-3)}$$