Solving Exponential and Logarithmic Equations

In section 3.4 you will learn to:

- Solve simple exponential and logarithmic equations.
- Solve more complex exponential and logarithmic equations.
- Use exponential and logarithmic equations to model and solve real-life problems.

An exponential equation has a variable in the exponent:

$$4^{x+1} = 16^{x-1}$$

To solve this one, make the bases alike.

Exponential functions are one-to-one, so when the bases are the same,the exponents are equal.

Some exponential equations can be solved by factoring:

$e^{2x} + e^{x} - 6 = 0$

More exponential equations: When solving for an exponent, it is often necessary to take the log of both sides of the equation.

a) $3^{t} = 8$

b) $2(3^{2t-5}) - 4 = 11$

c) $5^{1-x} = 12^{x}$

A **logarithmic equation** has a variable inside a logarithm. They typically require us to apply the properties of logarithms discussed in section 3.

a) $\log(1+3x) = 3$

b) $\ln x + \ln(x-3) = \ln(2x)$

Applications: Growth and Decay

Example 1:

A woman invests $25,000 in a savings account that compounds interest quarterly at a rate of 4%. How long will it take for the money to double?

Example 2:

The population of a colony of bacteria is measured to be 2000. Three hours later the population has grown to 3300. Assuming the population grows "exponentially" (compounding continuously) what should the population be after 10 hours?

Example 3:

The half-life of a certain isotope of radium is 1600 years. What percentage of a sample of the isotope will still be radioactive after 500 years?