

8.4 The determinant of a square matrix

In this section you will learn to
Find the determinants of square matrices.

A determinant is a unique scalar number associated with a square matrix.

For a 2 x 2 matrix:

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\det(A) = |A| =$$

$$|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} \\ = \boxed{ad - bc}$$

For a 3x3 matrix

$$B = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

$$\det(B) = |B| = \begin{vmatrix} \overset{1}{a} & \overset{2}{b} & \overset{3}{c} \\ d & e & f \\ g & h & i \end{vmatrix}$$

$$= a \begin{vmatrix} e & f \\ h & i \end{vmatrix} - b \begin{vmatrix} d & f \\ g & i \end{vmatrix}$$

$$+ c \begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

$$23 \text{ entry} \Rightarrow (-1)^{2+3} = (-1)^5 = -1$$

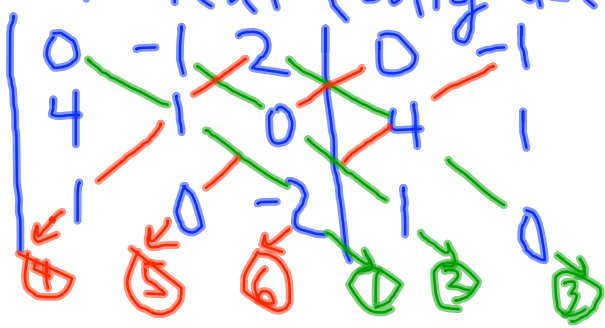
$$42 \Rightarrow (-1)^{4+2} = 1$$

Example 1: Find the determinant of each matrix.

a) $\begin{bmatrix} 3 & -2 \\ 4 & 6 \end{bmatrix} = B$ $\det(B) = |B| = 3(6) - (-2)(4)$
 $= 18 + 8 = 26$

b) $\begin{bmatrix} 0 & -1 & 2 \\ 4 & 1 & 0 \\ 1 & 0 & -2 \end{bmatrix} = C$ $|C| = (-1) \begin{vmatrix} 4 & 0 \\ 1 & -2 \end{vmatrix} + 1 \begin{vmatrix} 0 & 2 \\ 1 & -2 \end{vmatrix}$
 $- 0 \begin{vmatrix} 0 & 2 \\ 4 & 0 \end{vmatrix}$
 $= 1(-8 - 0) + 1(0 - 2) - 0$
 $= -8 - 2 = -10$

shortcut (only useful for 3x3 matrices)



$= 0 + 0 + 0$
 $- 2 - 0 - 8$
 $= -10$

c) What if we have a 4 x 4 matrix?

$$\begin{vmatrix} -1 & 3 & 0 & 1 \\ 2 & 0 & 4 & -3 \\ -2 & 1 & 7 & 0 \\ 3 & 2 & 0 & 5 \end{vmatrix} = 0 - 4 \begin{vmatrix} -1 & 3 & 1 \\ -2 & 1 & 0 \\ 3 & 2 & 5 \end{vmatrix} + 7 \begin{vmatrix} -1 & 3 & 1 \\ 2 & 0 & -3 \\ 3 & 2 & 5 \end{vmatrix} - 0$$

$$\begin{vmatrix} -1 & 3 & 1 & -1 & 3 \\ -2 & 1 & 0 & -2 & 1 \\ 3 & 2 & 5 & 3 & 2 \\ -1 & 3 & 1 & 1 & 3 \\ 2 & 0 & -3 & 2 & 0 \\ 3 & 2 & 5 & 3 & 2 \end{vmatrix}$$

$$= -5 + 0 + -4$$

$$- -30 - 0 - 3$$

$$= -12 + 30 = 18$$

$$0 + -27 + 4 - 30$$

$$-6 - 0 = -36$$

$$= -4(18) + 7(-36)$$

$$= -72 - 252$$

$$= -324$$



Example 2:

Solve for x.

$$\begin{vmatrix} x+4 & -2 \\ 7 & x-5 \end{vmatrix} = 0$$

$$(x+4)(x-5) - (-14) = 0$$

$$x^2 + 4x - 5x - 20 + 14 = 0$$

$$x^2 - x - 6 = 0$$

$$(x+2)(x-3) = 0$$

$$x+2=0 \quad \text{or} \quad x-3=0$$

$$x = -2 \quad \quad x = 3$$

