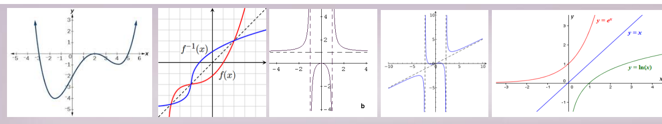

Math 1050 ~ College Algebra

10 Complex Zeros of Polynomials

$$-3x+4y=5$$
$$2x-y=-10$$
$$\begin{bmatrix}-3 & 4\\ 2 & -1\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}5\\ -10\end{bmatrix}$$

$$\sum_{k=1}^{m} k = \frac{m(m+1)}{2}$$
$$\sum_{k=0}^{n} z^k = \frac{1-z^{n+1}}{1-z}$$

Learning Objectives

- Perform operations on complex numbers.
- Find all complex zeros of a polynomial.
- Factor a polynomial to linear and irreducible quadratic factors.
- Use the conjugate of a complex zero to identify an additional zero.
- Create a polynomial given information that includes complex zeros.

Complex Numbers

The imaginary unit i satisfies the following properties:

- $i^2 = -1$
- If c is a real number, $c \geq 0$ then $\sqrt{-c} = (\sqrt{c})\cdot i$

A complex number is a number of the form $a + bi$ where a and b are real numbers and i is the imaginary unit.

The real numbers are a subset of the complex numbers.

The conjugate of a complex number, $a + bi$ is $a - bi$.

Expressed in symbols, $\overline{a+bi} = a - bi$.

Ex 1: Identify a, b and the conjugate of each of these complex numbers.

a) $-2 + 5i$ b) $6i$ c) 53 d) $\pi - i$

Arithmetic on these numbers is as expected.

EX 2: Perform these operations on complex numbers.

a) $(1-3i)+(2+5i)$ b) $(1-3i)(2+5i)$ c) $(1-3i)-(2+5i)$

d) $\dfrac{1-3i}{2+5i}$ e) $\sqrt{-3}\sqrt{-12}$ f) $\sqrt{(-3)(-12)}$

Ex 3: Perform this multiplication.

$$(x-(1+2i))(x-(1-2i))$$

<u>Complex Roots of Polynomial Functions</u>

The Fundamental Theorem of Algebra and Complex Factorization.

If f is a polynomial function with degree $n \geq 1$:

- f has at least one complex zero.
- In actuality, f has exactly n zeros, counting multiplicities.
- f has precisely n factors.

Furthermore:

- Complex zeros occur in conjugate pairs.
- Every polynomial can be factored into linear and quadratic factors with real coefficients.

Ex 4: Determine the complex zeros of $f(x) = 3x^2 - 2x + 2$.

Ex 5: Given $x + 3i$ is a factor of $f(x) = 2x^3 - 11x^2 + 18x - 99$, find all other zeros.

Ex 6: Use the techniques in this section and the last to find all the zeros of $f(x) = x^5 + 6x^4 + 10x^3 + 6x^2 + 9x$.