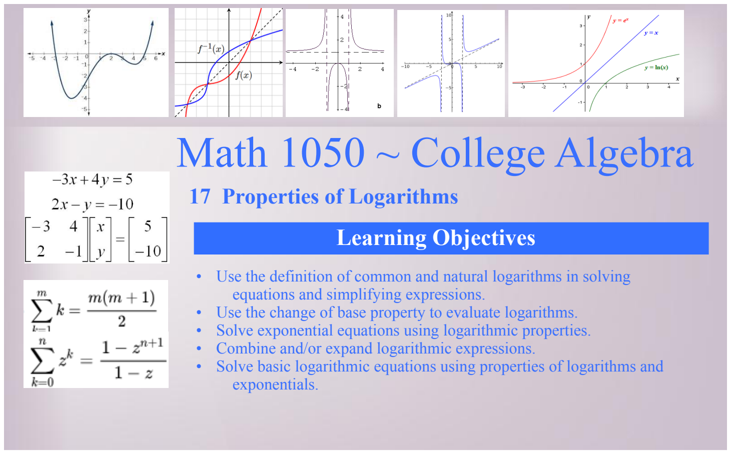


Common and Natural Logarithms

Base 10 is commonly used in logarithms. Thus, when no base is indicated, it is assumed to be base 10.

$\log x = \log_{10} x$

Ex 1: Evaluate these.

a) $\log 1{,}000{,}000$ b) $\log (10^{-3})$ c) $\log 0.01$ d) log (a trillion)

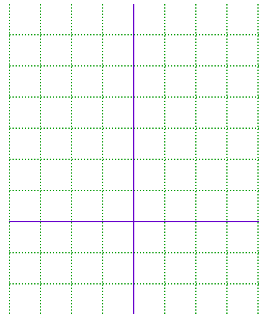
Another base is the irrational number, *e,* called the natural base. This is written using ln x.

$\log_e x = \ln x$

Ex 2: Evaluate these.

a) $\ln e$ b) $\ln e^{-3}$ c) $\ln e^{8}$ d) $\ln\left(\frac{1}{e^5}\right)$

Natural Exponential Base



$e \approx 2.718281828459....$

Ex 4: Sketch a graph of $y = e^x$.

The exponential base is used in financial and scientific calculations which we will explore in a later chapter.

Logarithm Properties

Let b be a positive number, not equal to 1, and let x be a positive number.

$$\log_b 1 = 0$$

$$\log_b b = 1$$

$$\log_b b^x = x$$

If $\log_b x = \log_b y$, then $x = y$

Ex 5: Evaluate these.

a) $\ln 1$ b) $\log 100$ c) $\ln e^{\pi}$ d) $\log (10^{0.2})$

Ex 6: Determine the value of x for each of these.

a) $\log x = \log (y + 5)$ b) $\ln x = \ln (\pi + 1)$

Properties of Logarithms

Change of Base Property

Let a and b be positive numbers, not equal to 1, and let x be a positive number.

$$\log_b x = \frac{\log_a x}{\log_a b}$$

Ex 7: True or false? $\log_2 3 = \frac{\log 3}{\log 2}$

Ex 8: Use your calculator to give an approximate value for these.

a) $\log_2 5$ b) log 50 c) ln 8 d) $\log_6 0.0002$

Inverse Properties

Let b be a positive number, not equal to 1.

$b^{\log_b x} = x$, for any positive number x

$\log_b b^x = x$, for any real number x

Ex 9: Use the inverse properties to simplify.

a) ln e - 2 b) $\log_5 1$ c) $6^{\log_6 20}$ d) $\log_3 3^{10}$

Exponent, Sum and Difference Properties of Logarithms

Let b be a positive number, not equal to 1, and let u and v be positive numbers.

$$\log_b(uv) = \log_b u + \log_b v$$

$$\log_b\left(\frac{u}{v}\right) = \log_b u - \log_b v$$

$$\log_b x^m = m\log_b x$$

Ex 10: Use these properties to expand these expressions.

a) $\log\sqrt{x^2(x+2)}$ b) $\ln\left(\frac{x^2-1}{x^3}\right)$, $x > 1$

Ex 11: Use these properties to contract these expressions into a single term.

a) $3\log x + 4\log y - 5\log z$ b) $\frac{1}{2}[\ln(x+1) + 2\ln(x-1)] - 6\ln x$