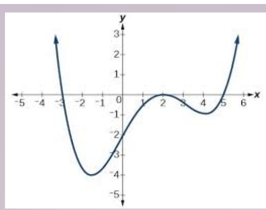
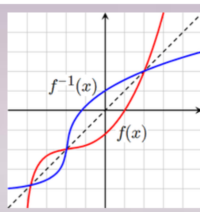
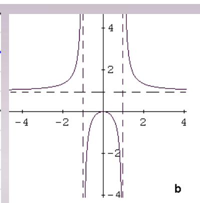
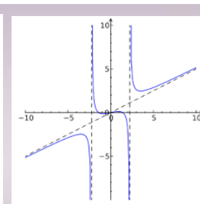
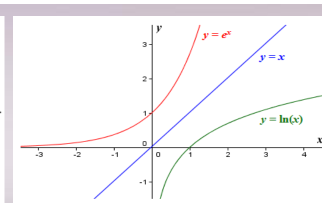










$$-3x + 4y = 5$$
$$2x - y = -10$$
$$\begin{bmatrix} -3 & 4 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -10 \end{bmatrix}$$

$$\sum_{k=1}^{m} k = \frac{m(m+1)}{2}$$
$$\sum_{k=0}^{n} z^k = \frac{1 - z^{n+1}}{1 - z}$$

Math 1050 ~ College Algebra

1 Introduction to Functions

Learning Objectives

- Determine whether a relation represents a function.
- Use the vertical line test to identify graphs of functions.
- Find the domain and range from the graph of a function.
- Find input and output values of a function.
- Find the domain from the equation of a function.

A **relation** is a set of ordered pairs. The set of first components of the ordered pairs is called the **domain** and the set of second components of the ordered pairs is called the **range**.

input value ‖ independent variable

output value ‖ dependent variable (depends on input)

Ex1: For each of these, state whether it is a relation, and if it is, list the elements in the domain and in the range.

a) {(1,5), (5,-2), (5,4), (3,2)}

Yes, it's a relation.

D: {1, 5, 3} R: {5, -2, 4, 2}

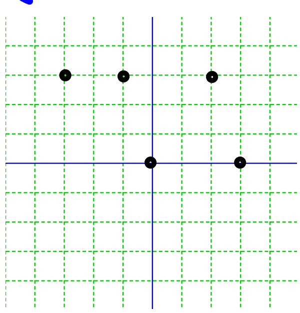
b)

inputs	outputs
Bud	15
May	16
Ezi	17
Zhu	18
Tia	19

D: {Bud, May, Ezi, Zhu, Tia}

R: {15, 16, 17, 18, 19}

c)



inputs D = {-3, -1, 0, 2, 3}

outputs R = {3, 0}

d) Input values: days of the week
Output values: final letter in word

D: {Sunday, Monday, Tuesday, Wed., Thurs., Fri., Sat.}

R: {y}

e) {name, rank, serial number}

not a relation

A **function** is a relation in which any two ordered pairs with the same first component also have the same second component.

a function has only one output for any given input

(fn means function)

Ex 2: From example 1, which of the relations are functions?

a) {(1,5), (5,-2), (5,4), (3,2)}

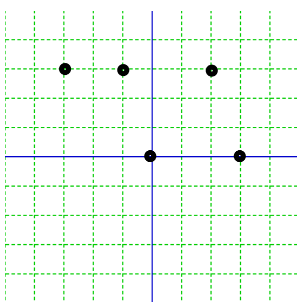
not a fn, because input 5 has 2 outputs

b)

Bud	15
May	16
Ezi	17
Zhu	18
Tia	19

yes, is a fn because every input has only one output

c)



Yes, it's a fn because every input has only one output

d) Input values: days of the week
Output values: final letter in word

ex (Tuesday, y)
(Wed., y)
Yes, a fn.

An equation in two variables can be a relation as can a 2-dimensional graph.

Ex 3: Which of these are functions?

y = output variable name

x = input variable name

a) $x+3 = y^2$

$y = \pm\sqrt{x+3}$ ⇒ there are 2 outputs for most inputs ⇒ it is <u>not</u> a fn.

b) $2y = \sqrt{x-1}$

$y = \frac{1}{2}\sqrt{x-1}$ ⇒ for every x-value, we get back one y-value ⇒ is a fn

c) $x^2 + y^2 = 9$

<u>ex</u> if $x=\sqrt{5}$, then $x^2=5$

$5+y^2=9 \iff y^2=4 \iff y=\pm 2$

So one particular x-value yielded two y-values ⇒ is <u>not</u> a fn

d) {(3,1), (2,1), (5,1), (6,2)}

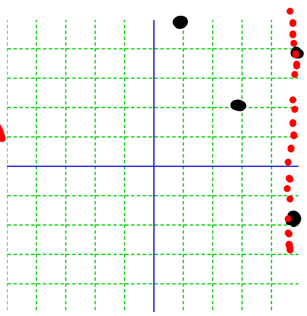
every input has only one output ⇒ this is a fn

<u>The Vertical Line Test</u>: A graph represents a function if no vertical line intersects it at more than one point.

Ex 4: Use the vertical line test to determine if these relations are functions.

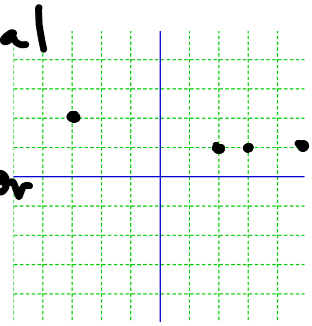
$R_1 = \{(1,5), (5,-2), (5,4), (3,2)\}$

this relation is <u>not</u> a fn.



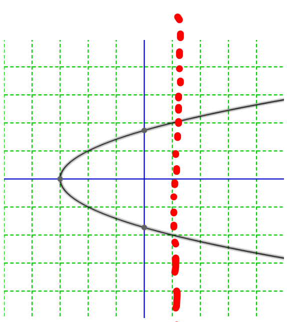
$R_2 = \{(3,1), (2,1), (5,1), (-3,2)\}$

all vertical lines go through either 0 or 1 pt



$\Rightarrow$ this is a fn.

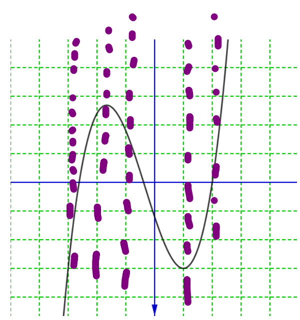
R_3



Some vertical lines pass through curve twice

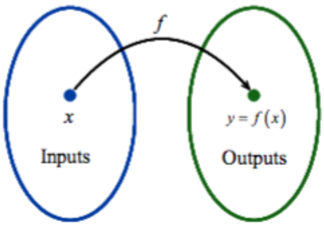
$\Rightarrow$ this relation is <u>not</u> a fn

R_4



this is a fn.

Function Notation



f is a fn that takes an input (x) and maps it to an output (y).

$y = f(x)$ (read "f of x")

Ex 5: Evaluate these functions for the given values.

a) $f(x) = \sqrt{x+8} + 2$

$f(-8) = \sqrt{-8+8} + 2 = 0 + 2 = 2$

$f(x-8) = \sqrt{(x-8)+8} + 2 = \sqrt{x} + 2$

$f(a) = \sqrt{a+8} + 2$

output when input=2

b) $g(2) = -3$

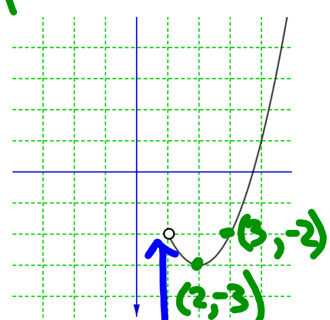
$g(0) =$

g(0) is undefined

$g(a) = -2$ for $a = 3$

output = -2, input?

$g(x)$



hole at (1,-2)

Domain of Functions

The domain of a function is the set of all input values for which the function is defined.

Implicit domain	Explicit domain
domain that's implied by computations needed in the function	handed to you

Ex 6: Determine the domain for each of these functions and identify as implicit or explicit.

a) $f(x)=\sqrt[3]{x+4}$

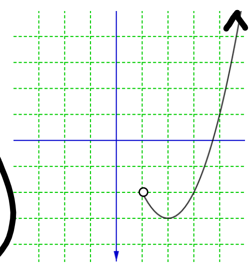
D: $x \in \mathbb{R}$ (implicit)

$\in$: element of

$\mathbb{R}$: set of real numbers

note: we can take cube root of any number

b) $p(x)$



D: $x>1$ (or $x \in (1,\infty)$) (explicit)

c) $g(x)=\dfrac{3}{x^2-2x} = \dfrac{3}{x(x-2)}$

D: $x \in \mathbb{R}$, $x \neq 0, 2$ (because those x-values make denominator zero)

(or $(-\infty,0)\cup(0,2)\cup(2,\infty)$)

(implicit)

d) $f(x)=\dfrac{\sqrt{x+4}}{4+x}$

① can't divide by zero $\Rightarrow x \neq -4$

and

② can only take square root of nonnegative #s

$\Rightarrow x+4 \geq 0$ (implicit)

$x \geq -4$

$\Rightarrow$ D: $x>-4$

(or $x \in (-4,\infty)$)

e) $h(x)=5x-3,\ \ x>-1$

D: $x>-1$ (or $(-1,\infty)$)

(explicit)