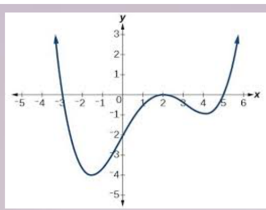
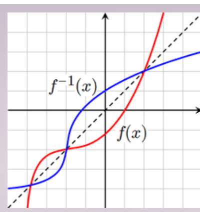
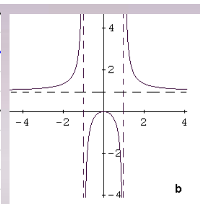
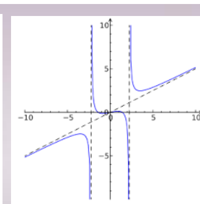
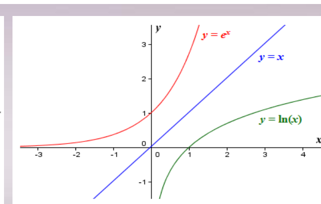










Math 1050 ~ College Algebra

22 Systems of Linear Equations and Applications

$$-3x + 4y = 5$$
$$2x - y = -10$$
$$\begin{bmatrix} -3 & 4 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -10 \end{bmatrix}$$

$$\sum_{k=1}^{m} k = \frac{m(m+1)}{2}$$
$$\sum_{k=0}^{n} z^k = \frac{1 - z^{n+1}}{1 - z}$$

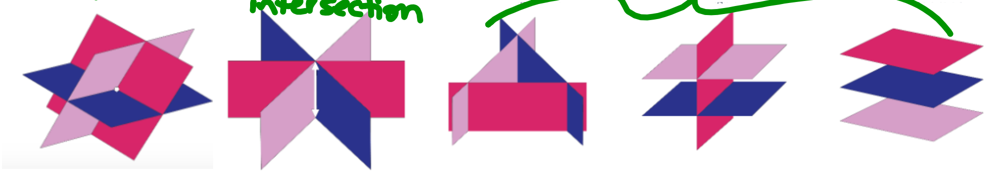
Learning Objectives

- Solve systems of three linear equations in three variables.
- Interpret solutions to 3×3 systems of linear equations.
- Solve applications of linear equations in three variables.

A **linear equation in three variables**, *x, y* and *z* is an equation of the form $\boxed{ax + by + cz = d}$ where *a,b,c* and *d* are constants and at least one of *a,b* and *c* is nonzero. Such an equation represents a plane in 3-D space.

Here are some possibilities of the intersection of three planes.

a pt. ① a line of intersection ② no point of intersection ③



We will solve these equations by using linear combinations. Your goal is to solve for *x, y* and *z*. This procedure is called **Elimination**.

Here are the legitimate actions you may take.

strategy for solving a system of linear eqns.

1. Exchange two rows.
2. Multiply a row by a nonzero constant.
3. Temporarily multiply a row by a nonzero constant and add it to another row, replacing either of those rows with the result.

Ex 1: Solve this system by using Elimination.

$$\begin{aligned} ① \quad x - y + z &= 4 \\ ② \quad x + 3y - 2z &= -3 \\ ③ \quad 3x + 2y + 2z &= 6 \end{aligned}$$

$2x-2y+2z=8$

$x-y+z=4$ (2) ↓ $\boxed{3}$

$x+3y-2z=-3$

$4x+5y=3$

aside

$$\begin{aligned} x+3y-\cancel{2z} &= -3 \\ +\ 3x+2y+\cancel{2z} &= 6 \\ \hline 4x+5y &= 3 \end{aligned}$$

$$\begin{aligned} x-y+z &= 4 \quad ① \\ 3x+y &= 5 \quad ② \\ 4x+5y &= 3 \quad ③ \end{aligned}$$

$-15x-5y=-25$

② $3x+y=5$ (-5)

③ $4x+5y=3$ ↓ $\boxed{3}$

② $3x+y=5$

③ $-11x=-22 \Leftrightarrow x=2$

② $3(2)+y=5 \Leftrightarrow y=-1$

① $x-y+z=4$

$2-(-1)+z=4$

$3+z=4$

$z=1$

Solution: $\boxed{(2,-1,1)}$ the three planes intersect in this one pt.

Ex 2: Solve
$$-3x+6y-3z=-12$$
$$\begin{aligned} x - 2y + z &= 4 \quad ① \quad (-3) \\ 3x - 6y + 3z &= 7 \quad ② \\ 2x + y + 4z &= 2 \quad ③ \end{aligned}$$

$x-2y+z=4$

$\boxed{0=-5}$ this is a problem

$2x+y+4z=2$

because $0 \neq -5$ (i.e. $0=-5$ is not a true statement), we know there is <u>no</u> pt of intersection.

N.S.

Ex 3: Solve
$$\begin{cases} x - 2y - z = -5 & ① \\ 2x + y + z = 5 & ② \end{cases}$$

(geometrically, we have only 2 planes which means they can intersect either in a line or not at all)

$x-2y-z=-5$ ①

$3x-y \quad =0$ ②

solve ② for y. $y=3x$ ②

substite into ①. $x-2(3x)-z=-5$

$-5x-z=-5$

$-5x=-5+z$

$5-5x=z$

$$\left.\begin{cases} x=x \\ y=3x \\ z=5-5x \end{cases}\right\} \text{line of intersection}$$

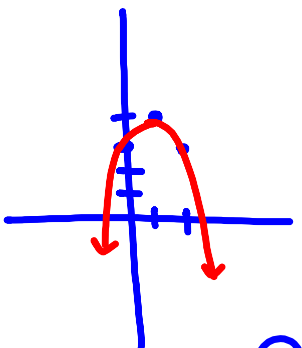
or we can write answer as

$(x, 3x, 5-5x)$

or $(t, 3t, 5-5t)$

Ex 4: Find the equation of the parabola, y = $ax^2 + bx + c$ that passes through these three points, (0,3), (1,4) and (2,3).

1 2 3



$$\left.\begin{array}{lll} (1) & 3=0+0+c & 3=c \\ (2) & 4=a(1)+b(1)+c & 4=a+b+c \\ (3) & 3=a(4)+b(2)+c & 3=4a+2b+c \end{array}\right\}$$

$$\begin{array}{ll} (2) & 4=a+b+3 \\ (3) & 3=4a+2b+3 \end{array} \Longleftrightarrow \begin{array}{l} 1=a+b \\ 0=4a+2b \end{array}$$

choose substitution method. (3) $2b=-4a$

$b=-2a$

(2) $1=a+(-2a)$

$1=-a$

$a=-1$ $\Rightarrow b=-2(-1)$

$b=2$

$\Rightarrow$ we have parabola

$$\boxed{y=-x^2+2x+3}$$