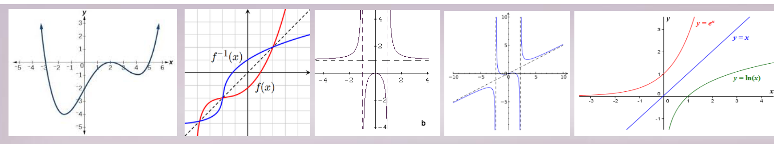

Math 1050 ~ College Algebra

$$-3x+4y=5$$
$$2x-y=-10$$
$$\begin{bmatrix} -3 & 4 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 5 \\ -10 \end{bmatrix}$$

$$\sum_{k=1}^{m} k = \frac{m(m+1)}{2}$$
$$\sum_{k=0}^{n} z^k = \frac{1-z^{n+1}}{1-z}$$

25 Systems of Linear Equations: Matrix Inverses

Learning Objectives

- Find the inverse of a 2×2 or a 3×3 matrix.
- Solve a system of linear equations using an inverse matrix.

Inverse Matrix

If A and B are square matrices, $n \times n$, such that $AB = BA = I_n$, then B is the inverse matrix of A and can be denoted as A^{-1}.

Ex 1: Show that B is A^{-1}. $A = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$ $B = \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}$

Process for finding an inverse matrix.

1. Augment A with I.

2. Perform row operations until the left side looks like I.

3. The right side will be A^{-1}.

Ex 2: Determine the inverse of each matrix, if it exists.

a) $\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$

b) $\begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ 6 & -2 & -3 \end{bmatrix}$

Let's derive a formula for the inverse of a 2×2 matrix.

We can write a system of linear equations as a matrix equation

$AX = C$, where A is a matrix of coefficients, X is the matrix of variables and C is the matrix of constants.

Ex 3: Write this system of equations as a matrix equation.

$2x + y = 4$

$5x + 3y = 6$

Ex 4: Using this fact from Ex. 1, $A = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix} \quad A^{-1} = \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}$
find the solution to Ex. 3.

Ex 5: Refer back to example 2 to solve these systems of equations.

a) $\begin{aligned} 2x+3y &= 0 \\ x+4y &= -5 \end{aligned}$

b) $\begin{aligned} x-y &= 2 \\ x-z &= 3 \\ 6x-2y-3z &= 15 \end{aligned}$

Ex 6: Solve this system using the techniques of this lesson.

$\begin{aligned} 2x-3y &= 8 \\ -4x+6y &= -5 \end{aligned}$