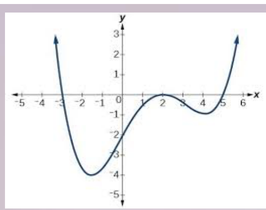
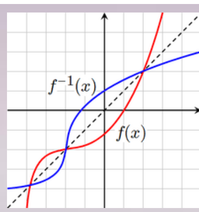
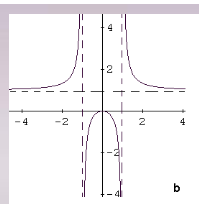
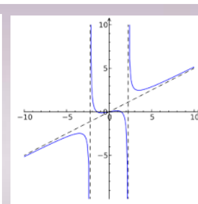
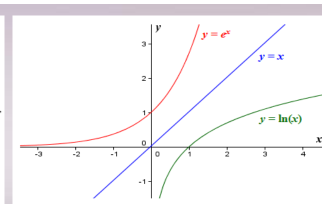










Math 1050 ~ College Algebra

$$-3x + 4y = 5$$
$$2x - y = -10$$
$$\begin{bmatrix} -3 & 4 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -10 \end{bmatrix}$$

$$\sum_{k=1}^{m} k = \frac{m(m+1)}{2}$$
$$\sum_{k=0}^{n} z^k = \frac{1 - z^{n+1}}{1 - z}$$

6.5 Supplemental Video

Learning Objectives

- Practice completing the square.
- Develop the quadratic formula.
- Develop the formula for the vertex of a quadratic function.

Completing the Square

For a good explanation of how to complete the square, see

http://www.mathsisfun.com/algebra/completing-square.html

This is useful in solving a quadratic equation and in putting that equation in standard form.

Ex 1: Solve by completing the square.

$\left(\frac{5}{2(2)}\right)^2 = \frac{25}{16}$

a) $x^2 - 6x - 3 = 0$

$(x^2 - 6x + 9) - 3 = 0$
-9

$\left(\frac{-6}{2}\right)^2 = 9$

$(x^2 - 6x + 9) - 12 = 0$

$(x-3)^2 - 12 = 0$

$(x-3)^2 = 12$

$(x-3) = \pm\sqrt{12}$

$x = 3 \pm \sqrt{12}$

$\boxed{x = 3 \pm 2\sqrt{3}}$

b) $3x^2 - 6x - 9 = 0$

$3(x^2 - 2x - 3) = 0$

$3(\underbrace{x^2 - 2x + 1} - 4) = 0$

$3((x-1)^2 - 4) = 0$

$(x-1)^2 - 4 = 0$

$(x-1)^2 = 4$

$x - 1 = \pm 2$

$x = 1 \pm 2$

$\boxed{x = 3, -1}$

c) $2x^2 - 5x + 4 = 0$

$2\left(x^2 - \frac{5}{2}x + 2\right) = 0$

$2\left(\underbrace{x^2 - \frac{5}{2}x + \frac{25}{16}} + 2 - \frac{25}{16}\right) = 0$

$2\left(\left(x - \frac{5}{4}\right)^2 + \frac{7}{16}\right) = 0$

$\left(x - \frac{5}{4}\right)^2 + \frac{7}{16} = 0$

$\left(x - \frac{5}{4}\right)^2 = -\frac{7}{16}$ this can't happen

N.S.

Ex 2: Put these equations in standard form. $y = a(x-h)^2 + k$

a) $y = x^2 + 2x - 2$

$y = \underbrace{x^2 + 2x + 1} - 2 - 1$

$\boxed{y = (x+1)^2 - 3}$

note: vertex $(-1, -3)$

b) $y = 2x^2 - 4x - 3$

$y = 2(x^2 - 2x) - 3$

$y = 2(x^2 - 2x + 1)$
$-3 - 2(1)$

$\boxed{y = 2(x-1)^2 - 5}$

c) $y = \underbrace{-\tfrac{1}{2}x^2 - 3x} + 5$

$y = -\frac{1}{2}(x^2 + 6x) + 5$

$y = -\frac{1}{2}(x^2 + 6x + 9)$
$+ 5 - 9\left(-\frac{1}{2}\right)$

$y = -\frac{1}{2}(x+3)^2 + 5 + \frac{9}{2}$

$\boxed{y = -\frac{1}{2}(x+3)^2 + \frac{19}{2}}$

Deriving the Quadratic Formula

If $ax^2 + bx + c = 0$, $a \neq 0$, then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Ex 3: Solve this equation for x, if a, b and c are constants. $(a \neq 0)$

$ax^2 + bx + c = 0$

$$a\left(\underline{x^2 + \frac{b}{a}x} + \frac{c}{a}\right) = 0$$

$$\left(\frac{\frac{b}{a}}{2}\right)^2 = \left(\frac{b}{2a}\right)^2 = \frac{b^2}{4a^2}$$

$$a\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} + \frac{c}{a} - \frac{b^2}{4a^2}\right) = 0$$

$$\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}\right) + \left(\frac{c \cdot 4a}{a \cdot 4a} - \frac{b^2}{4a^2}\right) = 0$$

$$\left(x + \frac{b}{2a}\right)^2 + \left(\frac{4ac - b^2}{4a^2}\right) = 0$$

$$\left(x + \frac{b}{2a}\right)^2 = -\frac{(4ac - b^2)}{4a^2}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

$$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$x + \frac{b}{2a} = \frac{\pm\sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

Deriving the Formula for the Vertex

The vertex of $f(x) = ax^2 + bx + c$ is at the point $\boxed{\text{vertex} \left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)}$

$a \neq 0$

y-intercept is at $(0, c)$

$c = ax^2 + bx + c$

$0 = ax^2 + bx$

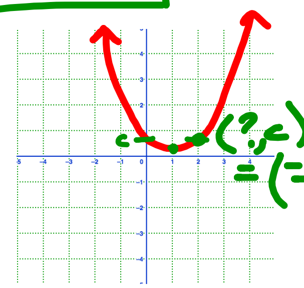
$0 = x(ax + b)$

$x = 0$ or $ax + b = 0$

$ax = -b$

$x = \frac{-b}{a}$

$\Rightarrow$ x-coord. of vertex is midpt of the two x-values 0 and $\frac{-b}{a}$ $\Rightarrow$ x-coord of vertex is $\frac{-b}{2a}$



$= \left(\frac{-b}{a}, c\right)$

Ex 4: Determine the vertex for each of these using the above method.

a) $y = x^2 + 2x - 2$

$a = 1, b = 2, c = -2$

x-coord of vertex

$x = \frac{-b}{2a} = \frac{-2}{2(1)} = -1$

$y = (-1)^2 + 2(-1) - 2$

$= 1 - 2 - 2 = -3$

vertex $\boxed{(-1, -3)}$

b) $y = 2x^2 - 6x - 3$

$a = 2, b = -6, c = -3$

$x = \frac{-(-6)}{2(2)} = \frac{6}{4}$

$= \frac{3}{2}$

$y = 2\left(\frac{3}{2}\right)^2 - 6\left(\frac{3}{2}\right) - 3$

$y = 2\left(\frac{9}{4}\right) - 9 - 3$

$y = \frac{9}{2} - 12 = \frac{-15}{2}$

vertex $\boxed{\left(\frac{3}{2}, \frac{-15}{2}\right)}$

c) $y = -\frac{1}{2}x^2 - 3x + 5$

$a = -\frac{1}{2}, b = -3, c = 5$

$x = \frac{-(-3)}{2\left(-\frac{1}{2}\right)} = \frac{3}{-1} = -3$

$y = -\frac{1}{2}(-3)^2 - 3(-3) + 5$

$y = -\frac{9}{2} + 9 + 5 = \frac{19}{2}$

vertex $\boxed{\left(-3, \frac{19}{2}\right)}$