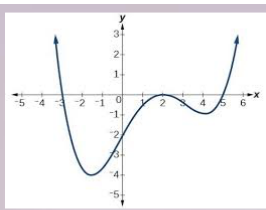

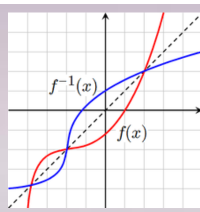

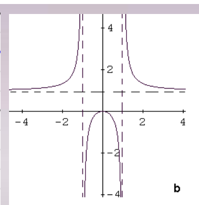

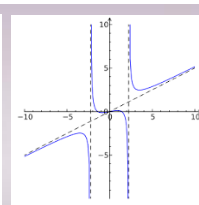

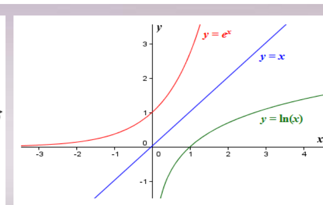

Math 1050 ~ College Algebra

$$-3x + 4y = 5$$
$$2x - y = -10$$
$$\begin{bmatrix} -3 & 4 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -10 \end{bmatrix}$$

$$\sum_{k=1}^{m} k = \frac{m(m+1)}{2}$$
$$\sum_{k=0}^{n} z^k = \frac{1 - z^{n+1}}{1 - z}$$

8 Using Synthetic Division to Factor Polynomials

Learning Objectives

- Use division to factor polynomials and determine zeros.
- Use synthetic division to simplify the division process.
- Use the Remainder Theorem to find function values of polynomials.
- Use the Factor Theorem to relate zeros to factors of polynomials.

When solving for the zeros of a function, it helps if we can break the function down into factors. Synthetic division will be useful to us.

$k \in \mathbb{R}$ (i.e. k is a constant)

Factor Theorem	**Remainder Theorem**
A polynomial $f(x)$ has a factor $(x-k)$ if and only if $f(k) = 0$.	If a polynomial $f(x)$ is divided by $(x-k)$, then the remainder $r = f(k)$.

k is a zero $\Leftrightarrow$ $(x-k)$ is factor

i.e. remainder value is same as value $f(k)$.

Note

Long division is ALWAYS useful for division of polynomials.

Synthetic division is only useful when dividing by $(x-k)$ where $k \in \Re$.

Ex 1: Use long division to divide $(4x^3 + 10x^2 - 2x - 5)$ by $(2x^2 - 1)$.

$$
\begin{array}{r}
2x+5 \\
2x^2-1 \enclose{longdiv}{4x^3+10x^2-2x-5} \\
-(\cancel{4x^3}+0x^2-2x) \\
\hline
10x^2+0x-5 \\
-(10x^2 \qquad -5) \\
\hline
0
\end{array}
$$

$\Rightarrow (4x^3+10x^2-2x-5) \div (2x^2-1)$
$= 2x+5$

Notice this process can help us factor the cubic (third degree) polynomial.

We get $(2x+5)(2x^2-1) = 4x^3 + 10x^2 - 2x - 5$

Remember long division of numbers:

$$
\begin{array}{r}
32 \\
71 \enclose{longdiv}{2304} \\
-213 \\
\hline
174 \\
-142 \\
\hline
32
\end{array}
$$

$\Rightarrow 2304 \div 71$
$= 32 + \frac{32}{71}$
$= 32\frac{32}{71}$

Ex 2: Divide $(x^3 + 4x^2 - 3x + 2)$ by $(x-3)$ in two ways. dividing by linear polynomial.

Long division

$$\begin{array}{r} x^2 + 7x + 18 \\ x-3 \overline{\smash{)}\, x^3 + 4x^2 - 3x + 2} \\ -(x^3 - 3x^2) \\ \hline 7x^2 - 3x + 2 \\ -(7x^2 - 21x) \\ \hline 18x + 2 \\ -(18x - 54) \\ \hline 56 \end{array}$$

remainder

$(x^3 + 4x^2 - 3x + 2) \div (x-3)$

$= \boxed{x^2 + 7x + 18 + \frac{56}{x-3}}$

Synthetic division

$x-3=0$

$x=3$

$$\begin{array}{c|cccc} 3 & 1 & 4 & -3 & 2 \\ & \downarrow & 3 & 21 & 54 \\ \hline & 1 & 7 & 18 & 56 \end{array}$$

remainder

$\Rightarrow (x^3 + 4x^2 - 3x + 2) \div (x-3)$

$= x^2 + 7x + 18 + \frac{56}{x-3}$

Ex 3: Use synthetic division to compute this quotient.

$(5x^3 + 6x + 8) \div (x + 2)$ $\quad$ $q(x)$ = quotient

Write the result in the form $f(x) = (x-k)(q(x)) + r(x)$ $\quad$ $r(x)$ = remainder

$$\begin{array}{r|rrrr} -2 & 5 & 0 & 6 & 8 \\ & \downarrow & -10 & 20 & -52 \\ \hline & 5 & -10 & 26 & -44 \end{array}$$

① ② ③ remainder

Tips to help w/ process:
① bring left # down
② multiply by root
③ in columns, add down

in our case $f(x) = 5x^3+6x+8$, $k=-2$

$q(x) = 5x^2-10x+26$, $r(x) = -44$

answer: $f(x) = (x+2)(5x^2-10x+26)-44$

Ex 4: Use the (a) remainder theorem and (b) synthetic division to show that (x+3) is a factor of this function.

$f(x) = 3x^3 + 5x^2 - 3x + 27$

$x+3=0$
$x=-3$

(b)

$$\begin{array}{r|rrrr} -3 & 3 & 5 & -3 & 27 \\ & \downarrow & -9 & 12 & -27 \\ \hline & 3 & -4 & 9 & 0 \end{array}$$

remainder

$\Rightarrow \dfrac{3x^3+5x^2-3x+27}{(x+3)} = 3x^2+-4x+9$

$\Rightarrow 3x^3+5x^2-3x+27 = (x+3)(3x^2-4x+9)$

(a) $f(-3) = 3(-3)^3+5(-3)^2-3(-3)+27$

$= 3(-27)+5(9)+9+27 = -81+45+36$

$= -81+81 = 0$ remainder

$\Rightarrow$ x+3 is factor of f(x) since the remainder is 0.

Ex 5: Use division to show that 2/3 is a solution of $48x^3 - 80x^2 + 41x - 6 = 0$. Use the result to factor the polynomial completely and find all solutions.

$$\begin{array}{c|cccc} \frac{2}{3} & 48 & -80 & 41 & -6 \\ & & 32 & -32 & 6 \\ \hline & 48 & -48 & 9 & 0 \end{array}$$

remainder

$\Rightarrow (x - \frac{2}{3})$ is factor of the cubic polynomial

$$48x^3 - 80x^2 + 41x - 6 = 0$$

$$(x - \tfrac{2}{3})(48x^2 - 48x + 9) = 0$$

$$(x - \tfrac{2}{3})(3)(16x^2 - 16x + 3) = 0$$

$$\underbrace{3(x - \tfrac{2}{3})}(4x - 1)(4x - 3) = 0$$

$$(3x - 2)(4x - 1)(4x - 3) = 0$$

$3x - 2 = 0$ or $4x - 1 = 0$ or $4x - 3 = 0$

$3x = 2$ $4x = 1$ $4x = 3$

$x = 2/3$ $x = 1/4$ $x = 3/4$