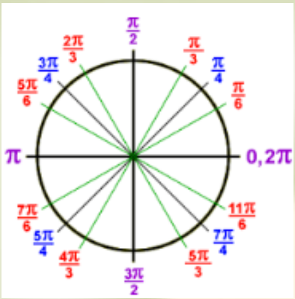
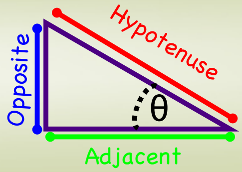
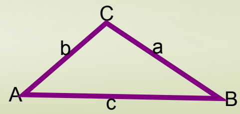
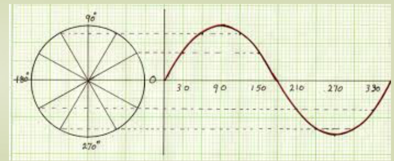








$\sin^2 u + \cos^2 u = 1$

$\sin 2u = 2 \sin u \cos u$

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$c^2 = a^2 + b^2 - 2ab\cos C$

Math 1060 ~ Trigonometry

10 Using Trigonometric Identities

Learning Objectives

In this section you will:

- **Learn and use the even/odd identities in simplifying trigonometric expressions and verifying identities.**
- **Learn the sum and difference identities for cosine, sine and tangent.**
- **Use the sum and difference identities to find values of trigonometric functions.**
- **Use the sum and difference identities in verifying trigonometric identities.**
- **Learn and apply the cofunction identities.**

Identities are useful in simplifying expressions and computing values for some of the less-familiar angles on the Unit Circle. In this video, we will familiarize ourselves with some important identities and use them to compute values, simplify expressions and verify other identities.

The Even/Odd Identities: For all applicable angles θ,		
• $\sin(-\theta) = -\sin(\theta)$ • $\csc(-\theta) = -\csc(\theta)$ odd	• $\cos(-\theta) = \cos(\theta)$ • $\sec(-\theta) = \sec(\theta)$ even	• $\tan(-\theta) = -\tan(\theta)$ • $\cot(-\theta) = -\cot(\theta)$ odd

Ex 1: Simplify these expressions.

a) sin(-x)cos(-x)sec(-x)

$= (-\sin x)(\cos(x))(\sec x)$

$= -\sin x \cancel{\cos x}\left(\frac{1}{\cancel{\cos x}}\right)$

$= -\sin x$

b) -cot(-x)tan(x)

(x in domain of both cot x & tan x)

$= -(-\cot x)\tan x$

$= \cot x \tan x$

$= 1$

Let's play a little True False game.

Ex 2: Identify these as True or False:

a) $5(c+d) = 5c + 5d$ True (distributivity)

b) $\frac{a+b}{2} = \frac{a}{2} + \frac{b}{2}$ True (")

c) $(x+y)^2 = x^2 + y^2$ False (exponents do <u>NOT</u> distribute

d) $\sqrt{p+q} = \sqrt{p} + \sqrt{q}$ False (") through addition)

e) $\sin(u+v) = \sin u + \sin v$

ex $u = v = \frac{\pi}{2}$. $\sin(u+v) = \sin \pi = 0$

$\sin u + \sin v = \sin(\pi/2) + \sin(\pi/2) = 1 + 1 = 2$

False (sine function does NOT distribute through addition.)

We can tell by inspection that *sin* 30º + *sin* 60º ≠ *sin* 90º.

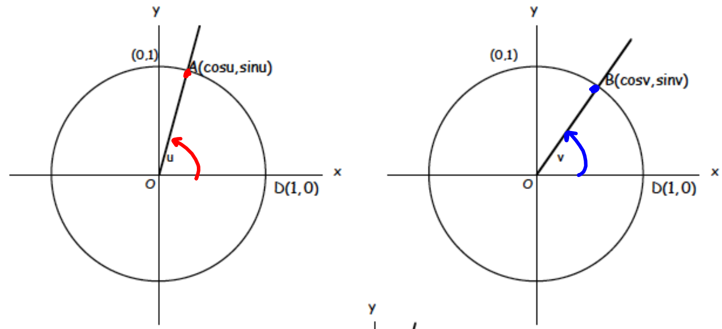
So, we need a set of sum/difference identities.

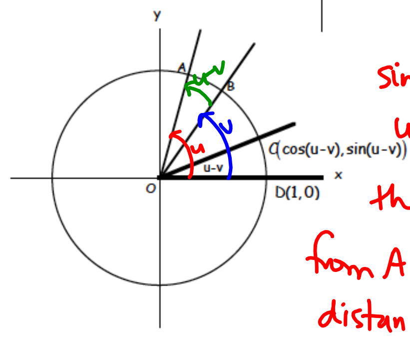
Sum and Difference Identities: For all applicable angles α and β,

- $\sin(\alpha \pm \beta) = \sin(\alpha)\cos(\beta) \pm \cos(\alpha)\sin(\beta)$
- $\cos(\alpha \pm \beta) = \cos(\alpha)\cos(\beta) \mp \sin(\alpha)\sin(\beta)$
- $\tan(\alpha \pm \beta) = \dfrac{\tan(\alpha) \pm \tan(\beta)}{1 \mp \tan(\alpha)\tan(\beta)}$

$\cos(\alpha + \beta) = \cos\alpha \cos\beta - \sin(\alpha) \sin\beta$

$\cos(\alpha - \beta) = \cos\alpha \cos\beta + \sin\alpha \sin\beta$





Since angle u-v = angle u-v, the distance from A to B = distance from C to D

Distance from A to B = Distance from C to D

$$\left(\sqrt{\underbrace{(\cos u - \cos v)^2}_{\text{difference of x-values}} + (\sin u - \sin v)^2}\right)^2 = \left(\sqrt{(\cos(u-v) - 1)^2 + (\sin(u-v) - 0)^2}\right)^2$$

$$(\cos u - \cos v)(\cos u - \cos v) + (\sin u - \sin v)(\sin u - \sin v)$$
$$= (\cos(u-v) - 1)(\cos(u-v) - 1) + \sin^2(u-v)$$

$$(\cos^2 u - 2\cos u\cos v + \cos^2 v) + (\sin^2 u - 2\sin u\sin v + \sin^2 v)$$
$$= \cos^2(u-v) - 2\cos(u-v) + 1 + \sin^2(u-v)$$

$$\underbrace{(\cos^2 u + \sin^2 u)}_{=1} + \underbrace{(\cos^2 v + \sin^2 v)}_{=1} - 2(\cos u\cos v + \sin u\sin v)$$
$$= \underbrace{(\cos^2(u-v) + \sin^2(u-v))}_{=1} - 2\cos(u-v) + 1$$

$$\cancel{2} - 2(\cos u\cos v + \sin u\sin v) = \cancel{2} - 2\cos(u-v)$$

$$\cancel{-2}(\cos u\cos v + \sin u\sin v) = \cancel{-2}\cos(u-v)$$

$$\boxed{\cos u\cos v + \sin u\sin v = \cos(u-v)}$$

Sum and Difference Identities for Sine. For all angles α and β,

1 • $\sin(\alpha+\beta)=\sin(\alpha)\cos(\beta)+\cos(\alpha)\sin(\beta)$

2 • $\sin(\alpha-\beta)=\sin(\alpha)\cos(\beta)-\cos(\alpha)\sin(\beta)$

Ex 3: Determine these exact values of sin 75º in two ways.

a) $\sin(75^\circ) = \sin(\underset{\alpha}{30^\circ} + \underset{\beta}{45^\circ})$

1 $= \sin(30^\circ)\cos(45^\circ) + \cos(30^\circ)\sin(45^\circ)$

$= \frac{1}{2}\left(\frac{\sqrt{2}}{2}\right) + \frac{\sqrt{3}}{2}\left(\frac{\sqrt{2}}{2}\right)$

$= \boxed{\dfrac{\sqrt{2}+\sqrt{6}}{4}}$

b) $\sin(75^\circ) = \sin(\underset{\alpha}{120^\circ} - \underset{\beta}{45^\circ})$

2 $= \sin(120^\circ)\cos(45^\circ) - \cos(120^\circ)\sin(45^\circ)$

$= \frac{\sqrt{3}}{2}\left(\frac{\sqrt{2}}{2}\right) - \left(-\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$= \boxed{\dfrac{\sqrt{6}+\sqrt{2}}{4}}$

Sum and Difference Identities: For all applicable angles α and β,

- $\sin(\alpha \pm \beta) = \sin(\alpha)\cos(\beta) \pm \cos(\alpha)\sin(\beta)$
- $\cos(\alpha \pm \beta) = \cos(\alpha)\cos(\beta) \mp \sin(\alpha)\sin(\beta)$
- $\tan(\alpha \pm \beta) = \dfrac{\tan(\alpha) \pm \tan(\beta)}{1 \mp \tan(\alpha)\tan(\beta)}$

Ex 4: Use the sum/difference identities to find the exact values of these.

a) $\cos\frac{5\pi}{12} = \cos\left(\frac{(8-3)\pi}{12}\right)$

$= \cos\left(\frac{8\pi}{12} - \frac{3\pi}{12}\right) = \cos\left(\frac{2\pi}{3} - \frac{\pi}{4}\right)$

$= \cos\left(\frac{2\pi}{3}\right)\cos\left(\frac{\pi}{4}\right) + \sin\left(\frac{2\pi}{3}\right)\sin\left(\frac{\pi}{4}\right)$

$= -\frac{1}{2}\left(\frac{\sqrt{2}}{2}\right) + \frac{\sqrt{3}}{2}\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{6}-\sqrt{2}}{4}$

b) $\sin\left(-\frac{7\pi}{12}\right) = \sin\left(\frac{(-3-4)\pi}{12}\right)$

$= \sin\left(\frac{-3\pi}{12} - \frac{4\pi}{12}\right)$

$= \sin\left(\frac{-\pi}{4} - \frac{\pi}{3}\right)$

$= \sin\left(\frac{-\pi}{4}\right)\cos\left(\frac{\pi}{3}\right) - \cos\left(\frac{-\pi}{4}\right)\sin\left(\frac{\pi}{3}\right)$

$= \frac{-\sqrt{2}}{2}\left(\frac{1}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$

$= \frac{-\sqrt{2}-\sqrt{6}}{4}$

Ex 5: Evaluate $\tan 165°$.

$\tan 165° = \tan(120° + 45°)$

$= \dfrac{\tan(120°) + \tan(45°)}{1 - \tan(120°)\tan(45°)}$

$= \dfrac{\frac{\sqrt{3}}{-1} + 1}{1 - (-\sqrt{3})(1)}$

$= \dfrac{1-\sqrt{3}}{1+\sqrt{3}}$

Ex 6: Verify this cofunction identity.

$$\sin\left(\frac{\pi}{2} - x\right) = \cos x$$

Identity is an eqn that's true for all allowed values of the variable.

$$\sin\left(\frac{\pi}{2} - x\right) \qquad \left(\begin{array}{c} \alpha = \frac{\pi}{2} \\ \beta = x \end{array}\right)$$

$$= \sin\left(\frac{\pi}{2}\right)\cos x - \cos\left(\frac{\pi}{2}\right)\sin x$$

$$= 1 \cdot \cos x - 0 \cdot \sin x$$

$$= \cos x \quad \blacksquare$$

Note: there are many cofunction identities. See the list in book!