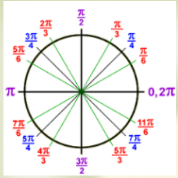

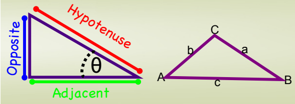

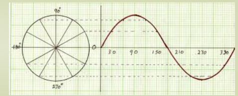

Math 1060 ~ Trigonometry

11 Multiple Angle Identities

$\sin^2 u + \cos^2 u = 1$

$\sin 2u = 2\sin u \cos u$

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$c^2 = a^2 + b^2 - 2ab\cos C$

Learning Objectives

In this section you will:

- **Learn the double and half angle identities for sine, cosine and tangent.**
- **Find trigonometric values of double and half angles.**
- **Verify identities involving double and half angles.**
- **Learn and apply the power reduction formulas for sine and cosine.**
- **Learn and apply product/sum formulas.**

The double angle identities are easy to generate using the identities for the sum of two angles.

$\sin(2\theta) = \sin(\theta + \theta)$

$\cos(2\theta) = \cos(\theta + \theta)$

$\tan(2\theta) = \tan(\theta + \theta)$

Double Angle Identities: For all applicable angles θ,

- $\sin(2\theta) = 2\sin(\theta)\cos(\theta)$
- $\cos(2\theta) = \begin{cases} \cos^2(\theta) - \sin^2(\theta) \\ 2\cos^2(\theta) - 1 \\ 1 - 2\sin^2(\theta) \end{cases}$
- $\tan(2\theta) = \dfrac{2\tan(\theta)}{1 - \tan^2(\theta)}$

Why do we need the double angle identities? Do they allow us to compute exact values of any angles?

- Simplify expressions.
- Solve equations with $2x$.

Ex 1: Solve this equation for values of x on the interval $[0,2\pi)$.

$\sin(2x) + \cos(x) = 0$

Starting with two forms of the double angle identity for the cosine, we can generate half-angle identities for the sine and cosine.

$\cos(2\theta) = 1 - 2\sin^2\theta$ $\qquad\qquad$ $\cos(2\theta) = 2\cos^2\theta - 1$

Power Reduction Formulas: For all angles θ,

- $\sin^2(\theta) = \dfrac{1 - \cos(2\theta)}{2}$
- $\cos^2(\theta) = \dfrac{1 + \cos(2\theta)}{2}$

$\sin\left(\dfrac{\theta}{2}\right)$ $\qquad\qquad$ $\cos\left(\dfrac{\theta}{2}\right)$

$\tan\left(\dfrac{\theta}{2}\right)$

Half Angle Formulas. For all applicable angles θ,

- $\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos(\theta)}{2}}$
- $\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1+\cos(\theta)}{2}}$
- $\tan\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos(\theta)}{1+\cos(\theta)}}$

where the choice of ± depends on the quadrant in which the terminal side of $\frac{\theta}{2}$ lies.

Ex 2: Use these identities to determine exact values.

a) sin (105º)

b) $\tan\left(\frac{7\pi}{12}\right)$

Ex 3: If θ is an obtuse angle and $\sin\theta = \frac{3}{5}$, find the exact value of these using double/half angle identities.

a) $\sin(2\theta)$

b) $\cos\left(\frac{\theta}{2}\right)$

c) $\tan(2\theta)$

Ex 4: Evaluate $\cos\left(\frac{7\pi}{12}\right)$ in two ways, using the half-angle identity and using the difference identity.