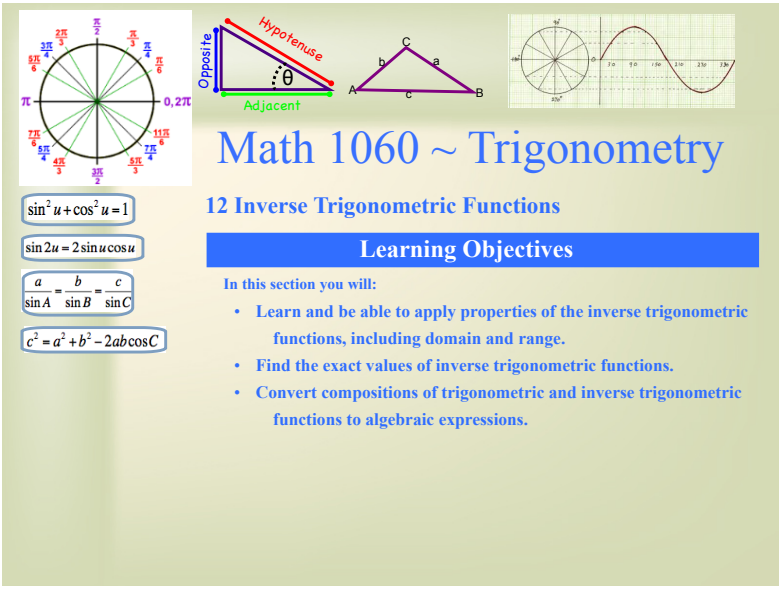
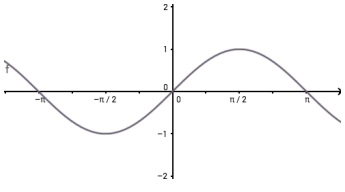


To find the inverse of the trigonometric functions, our first problem is that they are not one-to-one.

$y = \sin(x)$



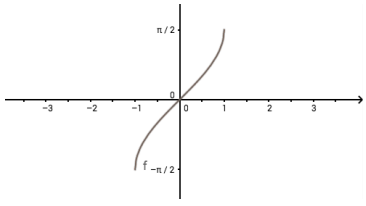
restricted domain:

range:

symmetry:

does $\sin(\sin^{-1}(x)) = x$?

$y = \sin^{-1}(x) = \arcsin(x)$



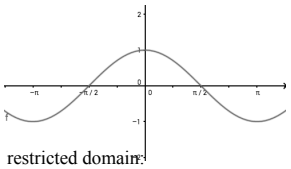
domain:

range:

symmetry:

does $\sin^{-1}(\sin(x)) = x$?

$y = \cos(x)$



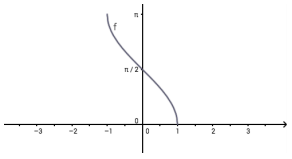
restricted domain:

range:

symmetry:

does $\sin(\cos^{-1}(x)) = x$?

$y = \cos^{-1}(x) = \arccos(x)$



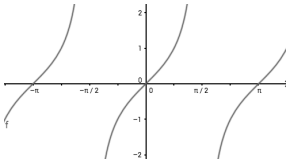
domain:

range:

symmetry:

does $\cos^{-1}(\cos(x)) = x$?

$y = \tan(x)$



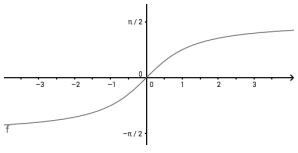
restricted domain:

range:

symmetry:

does $\tan(\tan^{-1}(x)) = x$?

$y = \tan^{-1}(x) = \arctan(x)$



domain:

range:

symmetry:

does $\tan^{-1}(\tan(x)) = x$?

When working these problems, it is easier if you think of the Unit Circle rather than the Cartesian graph.

$\sin^{-1}(x)$ answers will be in the interval $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

$\cos^{-1}(x)$ answers will be in the interval $[0,\pi]$

$\tan^{-1}(x)$ answers will be in the interval $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

Note that to compute the $\sec^{-1}(x)$, $\csc^{-1}(x)$ and $\cot^{-1}(x)$ you can turn each into a problem involving the three functions above.

Ex 1: Look at a Unit Circle and practice by finding the answers to these:

a) $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

b) $\cos^{-1}\left(\frac{\sqrt{2}}{2}\right)$

c) $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)$

d) $\sin^{-1}\left(-\frac{1}{2}\right)$

e) $\sec^{-1}\left(-\frac{2}{\sqrt{3}}\right)$

f) $\tan^{-1}(-1)$

Ex 2: Try these without looking at a Unit Circle.

a) $\sin^{-1}(-1)$

b) $\cos^{-1}(0)$

c) $\tan^{-1}(1)$

d) $\csc^{-1}(0)$

e) $\sec^{-1}(-2)$

f) $\cot^{-1}(-1)$

Ex 3: Which of these are true? Correct any that are false.

a) $\sin^{-1}\left(\sin\left(\frac{3\pi}{4}\right)\right)=\frac{3\pi}{4}$

b) $\cos\left(\cos^{-1}\left(\frac{1}{2}\right)\right)=\frac{1}{2}$

c) $\tan^{-1}(\tan\pi)=\pi$

Ex 4: These will require a bit more thought and perhaps a drawing of a triangle.
Evaluate these.

a) $\cos\left(\arctan\left(\frac{2}{3}\right)\right)$

b) $\tan\left(\sin^{-1}\left(\frac{3}{4}\right)\right)$

c) $\sec\left(\cos^{-1}\left(\frac{3x}{2}\right)\right)$

Ex 5: Evaluate these.

a) $\sec\left(\arctan\left(-\frac{3}{4}\right)\right)$

b) $\cot(\sin^{-1}(-0.2))$

Ex 5: Here is another challenge. Evaluate these.

a) $\sec\left(\arctan\left(-\frac{3}{4}\right)\right)$

b) $\cot(\sin^{-1}(-0.2))$

Ex 6: A plane flies at an altitude of 6 miles toward a point directly over an observer. Write the angle θ as a function of x, the horizontal distance from the observer to a point on the ground directly below the airplane.