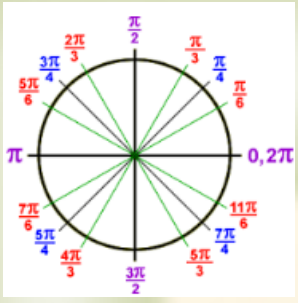
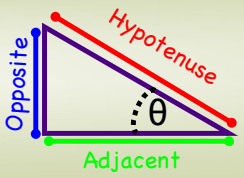
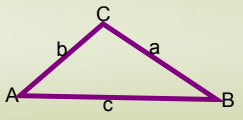
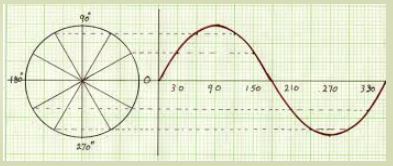








$\sin^2 u + \cos^2 u = 1$

$\sin 2u = 2\sin u \cos u$

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$c^2 = a^2 + b^2 - 2ab\cos C$

Math 1060 ~ Trigonometry

14 Solving Trigonometric Equations with Multiple Trigonometric Functions

Learning Objectives

In this section you will:

- **Write complete solutions to equations containing multiple trigonometric functions and/or arguments.**
- **Evaluate exact solutions in the interval [0,2π).**

In this section, we will solve more complicated trigonometric equations:

- those having different powers of the same function.
- those having multiple trigonometric functions.
- those containing multiple trigonometric functions and/or arguments.

Some identities from previous sections will come in handy for these.

Ex 1: Solve the equation $2\cos^2 x - \cos x = 0$ and list the solutions which lie in the interval $[0,2\pi)$.

REMEMBER: $\cos^2 x = (\cos x)^2$

$2\cos^2 x - \cos x = 0$

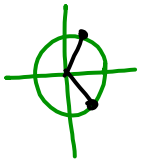
$\cos x(2\cos x - 1) = 0$

① $\cos x = 0$ OR ② $2\cos x - 1 = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$

$2\cos x = 1$

$\cos x = \frac{1}{2}$



$x = \frac{\pi}{3}, \frac{5\pi}{3}$

Ex 2: Solve the equation $\sec^2 x - 2\tan x = 4$.

Remember: $\tan^2 x + 1 = \sec^2 x$

$\sec^2 x - 2\tan x = 4$

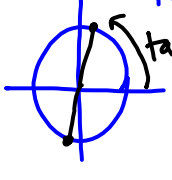
$\tan^2 x + 1 - 2\tan x = 4$ (treat this like quadratic eqn)

$\tan^2 x - 2\tan x - 3 = 0$

$(\tan x - 3)(\tan x + 1) = 0$

① $\tan x - 3 = 0$ or ② $\tan x + 1 = 0$

$\tan x = 3$

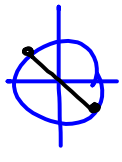


$x = \tan^{-1}(3)$

$(\tan^{-1}(3) + \pi)$

$x = \tan^{-1}(3) + n\pi$, $n \in \mathbb{Z}$

$\tan x = -1$



$x = \frac{3\pi}{4}, \frac{7\pi}{4}$

(going once around unit circle)

$x = \frac{3\pi}{4} + n\pi$, $n \in \mathbb{Z}$

Ex 3: State the solutions for these equations.

a) $\tan(2x) + \tan x = 0$

(use double angle id.)

$\tan(2x) = \frac{2\tan x}{1-\tan^2 x}$

eqn:

$\frac{2\tan x}{1-\tan^2 x} + \tan x = 0$

$\frac{2\tan x + \tan x(1-\tan^2 x)}{1-\tan^2 x} = 0$

$\frac{2\tan x + \tan x - \tan^3 x}{1-\tan^2 x} = 0$

$3\tan x - \tan^3 x = 0$

$\tan x(3-\tan^2 x) = 0$

① $\tan x = 0$ or ② $\tan^2 x = 3$

$x = 0, \pi$

$x = n\pi$, $n \in \mathbb{Z}$

$\tan x = \pm\sqrt{3}$

$x = \pm\frac{\pi}{3} + n\pi$

b) $\sin(2x)\sin x + \cos(2x)\cos x = 1$

(use double angle ids.)

$\sin(2x) = 2\sin x\cos x$

$\cos(2x) = 1 - 2\sin^2 x$

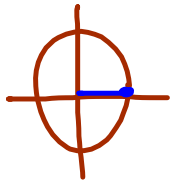
eqn:

$\sin(2x)\sin x + \cos(2x)\cos x = 1$

$2\sin x\cos x(\sin x) + (1-2\sin^2 x)\cos x = 1$

$2\sin^2 x\cos x + \cos x - 2\sin^2 x\cos x = 1$

$\cos x = 1$



$x = 0 + 2n\pi$

$x = 2n\pi$, $n \in \mathbb{Z}$