

Math 1060 ~ Trigonometry

16 Law of Cosines

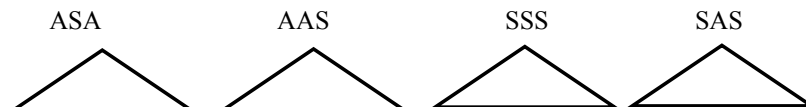
Learning Objectives

In this section you will:

- Use the Law of Cosines to solve oblique triangles.
- Solve SAS and SSS triangles.
- Use Heron's Formula to find the area of a triangle.
- Solve applied problems using the Law of Cosines and the Law of Sines.

$\sin^2 u + \cos^2 u = 1$
 $\sin 2u = 2 \sin u \cos u$
 $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
 $c^2 = a^2 + b^2 - 2ab \cos C$

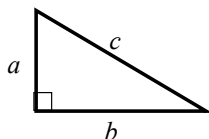
Congruence Postulates from Geometry



The Law of Cosines is just an adjustment to the Pythagorean Theorem which allows you to apply it to oblique triangles.

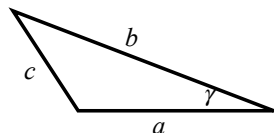
In a right triangle with hypotenuse length c ,

$$c^2 = a^2 + b^2$$



In any triangle with sides lengths of a, b, c ,

$$c^2 = a^2 + b^2 - 2ab \cos \gamma$$



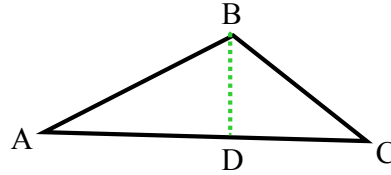
Proof of the Law of Cosines:

Given: $\triangle ABC$

Prove: $c^2 = a^2 + b^2 - 2ab \cos \gamma$

Draw altitude \overline{BD} to side \overline{AC} .

$BD = h$



As you work these, write the postulate which applies, SSS, SAS, ASA, AAS.

Ex 1: Triangle ABC has $a = 15$ cm, $b = 12$ cm, and γ measures 85° . Solve for the missing parts.

Ex 2: Find the angles in a triangle with sides of 6 m, 9 m and 11 m.

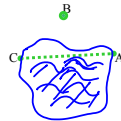
There is one more interesting formula for the area of a triangle given the three sides.

Heron's formula: $A = \sqrt{s(s-a)(s-b)(s-c)}$, $s = \frac{a+b+c}{2}$

The strategy for solving any triangle, given three parts:

- Draw the triangle.
- Label the parts.
- Determine which law to use.
- Solve.

Ex 3: A surveyor is measuring the width of a lake. He stands at point A and walks 50 m to point B, turns counter-clockwise 85° and walks 75 m to point C. How wide is the lake?



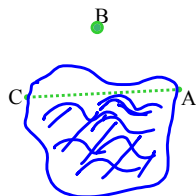
Ex 4: Find the area of a triangle with sides 7", 12", and 13" in two ways:

a) Use $A = \frac{1}{2}ab\sin\gamma$ and Heron's formula.

$$A = \frac{1}{2}ab\sin\gamma$$

$$\sqrt{s(s-a)(s-b)(s-c)}, \quad s = \frac{a+b+c}{2}$$

Ex 5: To estimate the dimensions of a lake, a surveyor starts at point A, walks 100 m to a tree at point B, turns 75° clockwise and measures the walk to point C as 70 m. What is the width of the lake from A to C?



Trigonometry and Bearings

In surveying and navigation, directions are often given in terms of bearings. This can be in one of two ways.

- a) Expressed as some east or west angle from north or south.
- b) Expressed as degrees in a clockwise direction from north.

Ex 5: Sketch each bearing and express it in both ways.

a) N 50° W



b) S 30° E



c) 200°



d) 320°



Ex 6: A plane flies due north for 200 miles, then turns to a bearing of 50° and flies 120 miles. How far is it from the starting point?