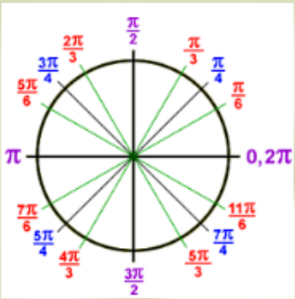
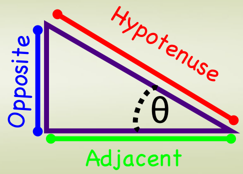
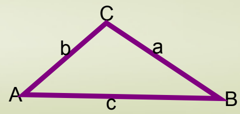
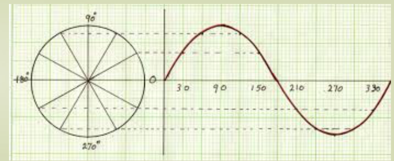








Math 1060 ~ Trigonometry

19 Trigonometric Representation of Complex Numbers

$\sin^2 u + \cos^2 u = 1$

$\sin 2u = 2\sin u \cos u$

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$c^2 = a^2 + b^2 - 2ab\cos C$

Learning Objectives

In this section you will:

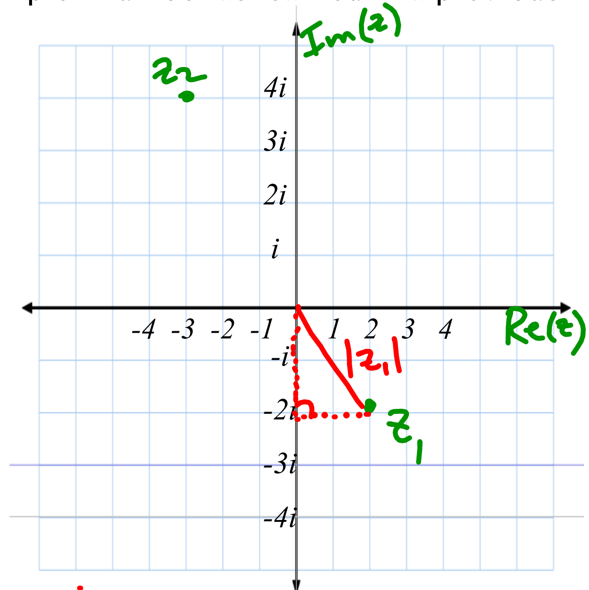
- Find the real part, the imaginary part, and the modulus of a complex number.
- Graph complex numbers.
- Convert between rectangular form and trigonometric form of complex numbers.

Review of Complex Numbers

What is i ? $i^2 = -1$ $i = \sqrt{-1}$ (imaginary #)

The rectangular form of a complex number is $\boxed{z = a + bi}$ where a is the real part and b is the imaginary part. This is represented by $Re(z) = a$ and $Im(z) = b$.

This exercise should serve as a review of complex number as learned in a previous course.



Ex 1: Let $z_1 = 2 - 2i$ and $z_2 = -3 + 4i$.

a) Sketch z_1 and z_2 in the complex plane

b) $z_1 + z_2 =$ $2-2i+-3+4i$
$= -1+2i$

c) $z_1 \times z_2 =$ $(2-2i)(-3+4i)$
$= -6+8i+6i-8i^2$
$= -6+14i-8(-1) = 2+14i$

d) $\overline{z_1} =$ conjugate of z_1
$= 2+2i$

$\overline{z_2} =$ conjugate of z_2
$= -3-4i$

e) $|z_1| =$ magnitude/abs.value/ modulus of z_1
$= \sqrt{2^2+(-2)^2} = \sqrt{8} = \sqrt{4 \cdot 2} = 2\sqrt{2}$

$|z_2| =$ $\sqrt{(-3)^2+4^2} = \sqrt{25} = 5$

f) $(z_1)^2 =$ $(2-2i)^2 = (2-2i)(2-2i)$
$= 4-4i-4i+4i^2 = 4-8i+(-4) = -8i$

You may be asking what is the square root of i?

Trigonometric Form (Polar Form) of a Complex Number

$\boxed{z = a + bi}$ becomes $\boxed{z = r(\cos\theta + i \sin \theta)} = r \text{ cis } \theta$.

- $r = |z|$ and is called the modulus of z.
- θ is called the argument of z, and $\tan\theta = \dfrac{b}{a}$.

  θ is the angle when sketched in standard position, on the interval $[0,2\pi)$.

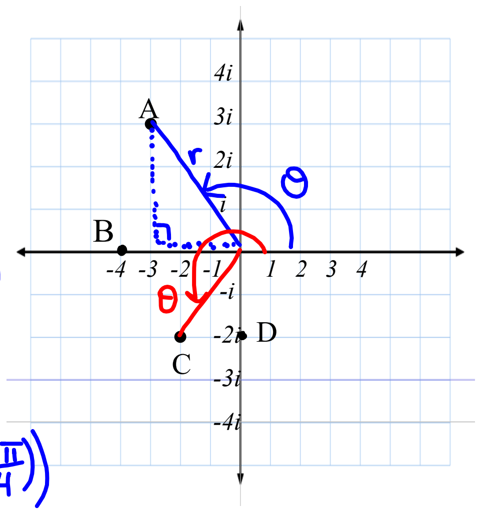
  $\tan^{-1}\left|\dfrac{b}{a}\right|$ will give you the reference angle.

  It is up to you to name the argument in the correct quadrant.

Note that the argument and the modulus are both positive.

$z = a + bi$ $\qquad$ $r = \sqrt{a^2 + b^2}$

Ex 2: State the coordinates of these points in rectangular form ($a + bi$) and in polar form (r cis θ) using radians.



A: $-3 + 3i = z$

$r = \sqrt{(-3)^2 + 3^2} = \sqrt{18} = 3\sqrt{2}$

$\tan\theta = \frac{3}{-3} = -1$ $\left(\text{note: } \theta' = \frac{\pi}{4}\right)$

$\theta = \frac{3\pi}{4}$

$z = 3\sqrt{2} \text{ cis}\left(\frac{3\pi}{4}\right)$

$= 3\sqrt{2}\left(\cos\left(\frac{3\pi}{4}\right) + i\sin\left(\frac{3\pi}{4}\right)\right)$

B: $z = -4 + 0i$

$r = \sqrt{(-4)^2 + 0^2} = 4$

$\tan\theta = \frac{0}{-4} = 0$

$\theta = \pi$

$z = 4 \text{ cis } \pi$

C: $z = -2 + -2i$

$r = \sqrt{(-2)^2 + (-2)^2} = \sqrt{8} = 2\sqrt{2}$

$\tan\theta = \frac{-2}{-2} = 1$ $\left(\text{note: } \theta' = \frac{\pi}{4}\right)$

$\theta = \frac{5\pi}{4}$

$z = 2\sqrt{2} \text{ cis}\left(\frac{5\pi}{4}\right)$

$z = 0 + -2i = -2i$

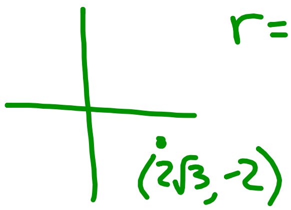
D: $r = 2$

$\theta = \frac{3\pi}{2}$

$z = 2 \text{ cis}\left(\frac{3\pi}{2}\right)$

Ex 3: Put these in trigonometric (polar) form, $r(\cos\theta + i\sin\theta)$.

a) $z_1 = 2\sqrt{3} - 2i$ (radians)



$r = \sqrt{(2\sqrt{3})^2 + (-2)^2}$

$= \sqrt{4\cdot 3 + 4}$

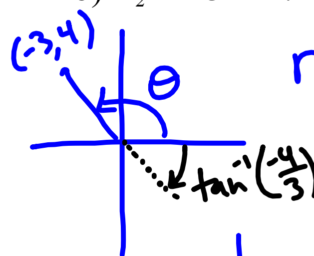
$= \sqrt{16} = 4$

$\tan\theta = \frac{-2}{2\sqrt{3}} = \frac{-1}{\sqrt{3}} = \frac{-1/2}{\sqrt{3}/2}$

$\theta = \frac{-\pi}{6} + 2\pi = \frac{11\pi}{6}$

$z = 4\,\mathrm{cis}\left(\frac{11\pi}{6}\right) = 4\left(\cos\left(\frac{11\pi}{6}\right) + i\sin\left(\frac{11\pi}{6}\right)\right)$

b) $z_2 = -3 + 4i$ (degrees)



$r = \sqrt{(-3)^2 + 4^2}$

$= \sqrt{25} = 5$

$\tan\theta = \frac{4}{-3}$

$\theta = \tan^{-1}\left(\frac{-4}{3}\right) + \pi$

$\theta = \tan^{-1}\left(\frac{-4}{3}\right) + 180°$

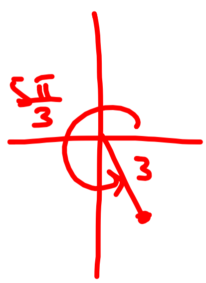
$\theta \approx 126.9°$

$z_2 \approx 5\,\mathrm{cis}(126.9°)$

Ex 4: Write these in rectangular form, $(a + bi)$.

a) $z_1 = 3\left(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right)$

$r = 3 \qquad \theta = \frac{5\pi}{3}$

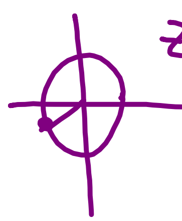


$z_1 = 3\left(\frac{1}{2} + i\left(\frac{-\sqrt{3}}{2}\right)\right)$

$z_1 = \frac{3}{2} - \frac{3\sqrt{3}}{2}i$

b) $z_2 = 20(\cos 210° + i\sin 210°)$

$z_2 = 20\left(-\frac{\sqrt{3}}{2} + i\left(\frac{-1}{2}\right)\right)$



$z_2 = -10\sqrt{3} - 10i$