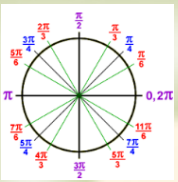
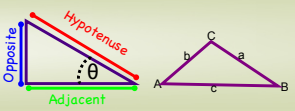
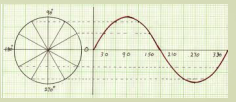






Math 1060 ~ Trigonometry

$\sin^2 u + \cos^2 u = 1$

$\sin 2u = 2\sin u \cos u$

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$c^2 = a^2 + b^2 - 2ab\cos C$

19 Trigonometric Representation of Complex Numbers

Learning Objectives

In this section you will:

- **Find the real part, the imaginary part, and the modulus of a complex number.**
- **Graph complex numbers.**
- **Convert between rectangular form and trigonometric form of complex numbers.**

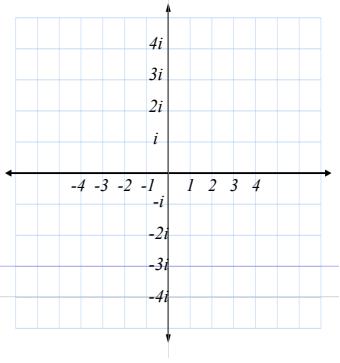
Review of Complex Numbers

What is i ?

The rectangular form of a complex number is $z = a + bi$, where a is the real part and b is the imaginary part. This is represented by $Re(z) = a$ and $Im(z) = b$.

This exercise should serve as a review of complex numbers as learned in a previous course.

Ex 1: Let $z_1 = 2 - 2i$ and $z_2 = -3 + 4i$.



a) Sketch z_1 and z_2 in the complex plane

b) $z_1 + z_2$ =

c) $z_1 \times z_2$ =

d) $\overline{z_1}$ = $\overline{z_2}$ =

e) $|z_1|$ = $|z_2|$ =

f) $(z_1)^2$ =

You may be asking what is the square root of i?

Trigonometric Form (Polar Form) of a Complex Number

$z = a + bi$ becomes $z = r(\cos\theta + i \sin\theta) = r\,cis\,\theta$.

- $r = |z|$ and is called the modulus of z.
- θ is called the argument of z, and $\tan\theta = \frac{b}{a}$.

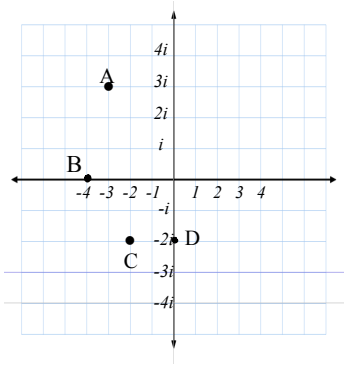
  θ is the angle when sketched in standard position, on the interval $[0,2\pi)$.

  $\tan^{-1}\left|\frac{b}{a}\right|$ will give you the reference angle.

  It is up to you to name the argument in the correct quadrant.

Note that the argument and the modulus are both positive.

Ex 2: State the coordinates of these points in rectangular form ($a + bi$) and in polar form (r cis θ) using radians.



Ex 3: Put these in trigonometric (polar) form, $r(\cos\theta + i \sin\theta)$.

a) $z_1 = 2\sqrt{3} - 2i$ (radians)

b) $z_2 = -3 + 4i$ (degrees)

Ex 4: Write these in rectangular form, $(a + bi)$.

a) $z_1 = 3\left(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right)$

b) $z_2 = 20(\cos 210° + i\sin 210°)$