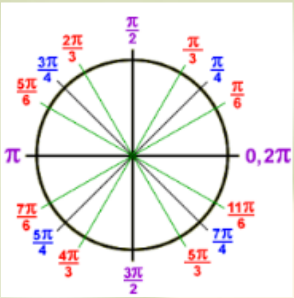
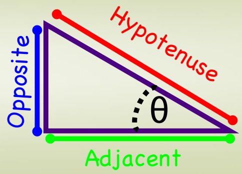
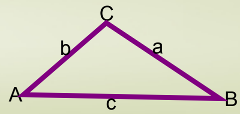
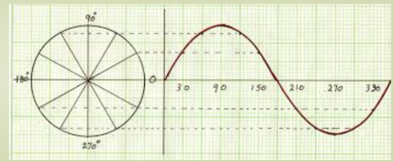








Math 1060 ~ Trigonometry

22 The Unit Vector and Vector Applications

$\boxed{\sin^2 u + \cos^2 u = 1}$

$\boxed{\sin 2u = 2\sin u \cos u}$

$\boxed{\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}}$

$\boxed{c^2 = a^2 + b^2 - 2ab\cos C}$

Learning Objectives

In this section you will:

- **Use vectors in component form to solve applications.**
- **Find the unit vector in a given direction.**
- **Perform operations on vectors in terms of i and j.**
- **Use vectors to model forces.**

The Unit Vector

$\hat{v}$ = unit vector

It is often useful in solving problems to find a vector in the same direction as the given vector, but of magnitude 1. This is called a unit vector. If $\|\boldsymbol{v}\| = 1$, then $\hat{\boldsymbol{v}}$ is a unit vector.

in direction of $\vec{v}$

If $\boldsymbol{v}$ is a nonzero vector, then $\dfrac{\boldsymbol{v}}{\|\boldsymbol{v}\|}$ is a unit vector in the direction of $\boldsymbol{v}$.

$$\boxed{\hat{v} = \frac{\vec{v}}{\|\vec{v}\|}}$$

Ex 1: Find a unit vector in the direction of $\boldsymbol{v} = \langle 3,-4 \rangle$.

$$\hat{v} = \frac{\langle 3,-4\rangle}{5} = \frac{1}{5}\langle 3,-4\rangle$$

$$\hat{v} = \left\langle \frac{3}{5}, \frac{-4}{5} \right\rangle$$

$$\|\vec{v}\| = \sqrt{3^2+(-4)^2} = \sqrt{25} = 5$$

The Principal Unit Vectors

It is also useful to have names for the principal unit vectors.

- The vector $\hat{\boldsymbol{i}}$ is defined by $\hat{\boldsymbol{i}} = \langle 1,0 \rangle$. $\longrightarrow \hat{i}$
- The vector $\hat{\boldsymbol{j}}$ is defined by $\hat{\boldsymbol{j}} = \langle 0,1 \rangle$ $\uparrow \hat{j}$

$$\langle a,b\rangle = a\hat{i} + b\hat{j}$$

This gives us another way to describe vectors. Thus $\langle a,b \rangle$ may also be $a\hat{\boldsymbol{i}} + b\hat{\boldsymbol{j}}$.

Ex 2: Describe the vector from example 1 in terms of unit vectors.

$$\vec{v} = \langle 3,-4\rangle = 3\hat{i} - 4\hat{j}$$

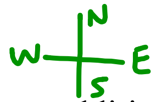
$$\hat{v} = \left\langle \frac{3}{5}, \frac{-4}{5}\right\rangle = \frac{3}{5}\hat{i} - \frac{4}{5}\hat{j}$$

Applications of Vectors

Vectors are useful in airplane navigation.

Ex 3: A plane is flying N 30º E at 400 mph and the wind is blowing west at 40 mph. What is the effective direction and speed of the plane?

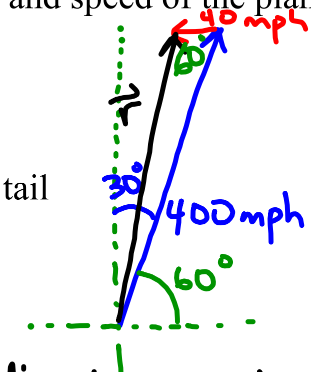
a) Draw a picture.



b) Place the vectors for proper addition.

c) Remember: the resultant is from the tail of the first to the tip of the last.

d) Now do the math.

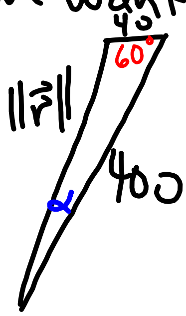


$\vec{p}$ = vector for plane

$\vec{w}$ = vector for wind

$\vec{r}$ = resultant vector

we want $\|\vec{r}\|$ and the direction angle



we have SAS case $\Rightarrow$ use law of Cosines

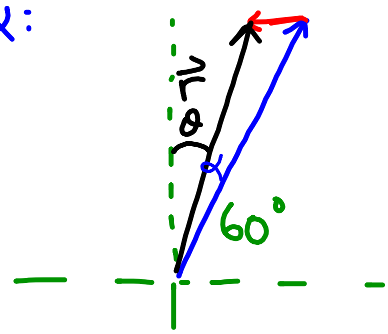
$$\|\vec{r}\|^2 = 400^2 + 40^2 - 2(400)(40)\cos 60^\circ$$
$$= 145{,}600$$
$$\|\vec{r}\| = \sqrt{145600} \simeq 381.6 \text{ mph}$$

use law of Sines to find α:

$$\frac{\sin\alpha}{40} \simeq \frac{\sin 60^\circ}{381.6}$$

$$\sin\alpha \simeq \frac{40\left(\frac{\sqrt{3}}{2}\right)}{381.6}$$

$$\alpha \simeq 5.2^\circ$$

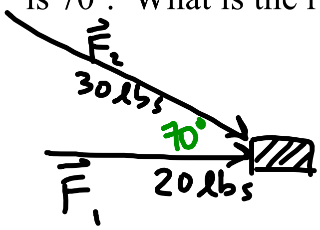


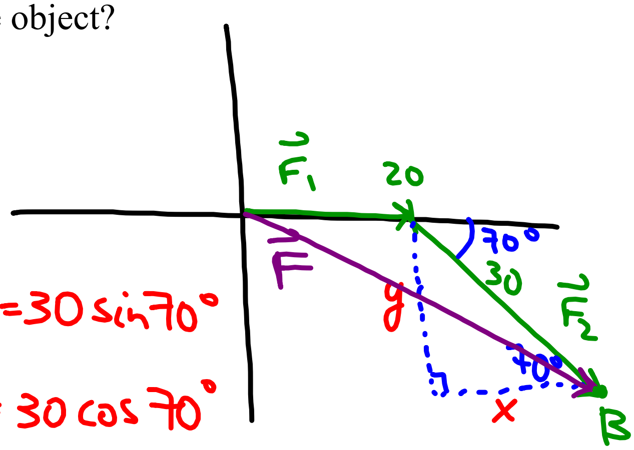
$\theta + \alpha = 30^\circ \Rightarrow \theta \simeq 30^\circ - 5.2^\circ = 24.8^\circ$

$\Rightarrow$ plane flies at ~381.6 mph N24.8°E

Vectors may be used to analyze forces acting on an object.

Ex 4: Two forces are pushing on an object, one exerts 30 lbs of pressure and a second exerts 20 lbs of pressure. The angle between the two forces is 70º. What is the resultant force on the object?





$\sin 70° = \frac{y}{30} \Leftrightarrow y = 30 \sin 70°$

$\cos 70° = \frac{x}{30} \Leftrightarrow x = 30 \cos 70°$

$\vec{F} = \vec{F}_1 + \vec{F}_2$

pt B: $(20 + 30\cos 70°, -30 \sin 70°)$

$\vec{F}$ = resultant force vector acting on the object

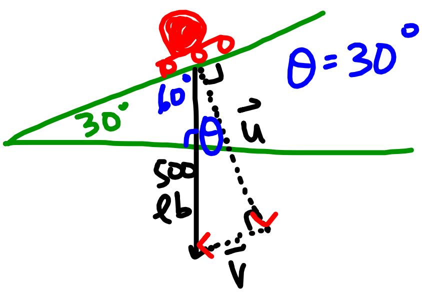
$= \langle 20 + 30\cos 70°, -30\sin 70° \rangle$

$\simeq \langle 30.26, -28.19 \rangle$

$\|\vec{F}\|$ is measured in lbs.

$\|\vec{F}\| \simeq \sqrt{30.26^2 + (-28.19)^2} \simeq 41.36$ lbs

Ex 5: A 500-lb rock is being wheeled up a 30 degree ramp. What force is necessary to keep it from rolling back down the ramp? What is the weight the ramp is actually supporting?



$\vec{u} + \vec{v} =$ the 500-lb gravity/weight vector

$\|\vec{u}\| =$ wt that the ramp supports

$\|\vec{v}\| =$ wt pushing against rock along ramp (to keep it from rolling down)

$\cos\theta = \dfrac{\|\vec{u}\|}{500}$

$\|\vec{u}\| = 500\cos 30° = 500\left(\frac{\sqrt{3}}{2}\right) = 250\sqrt{3} \simeq 433.01$ lbs

$\|\vec{v}\| = 500\sin 30° = 500\left(\frac{1}{2}\right) = 250$ lbs