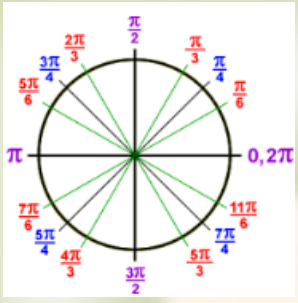
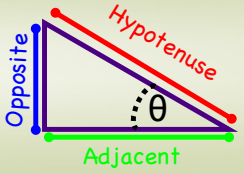
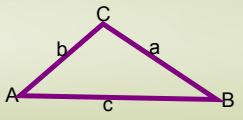
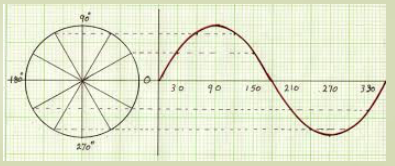








Math 1060 ~ Trigonometry

$\sin^2 u + \cos^2 u = 1$

$\sin 2u = 2\sin u \cos u$

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$c^2 = a^2 + b^2 - 2ab\cos C$

24 Curves Described by Parametric Equations

Learning Objectives

In this section you will:

- **Graph plane curves described by parametric equations.**
- **Analyze behavior in the graphs of parametric equations.**

Curves Described by Parametric Equations

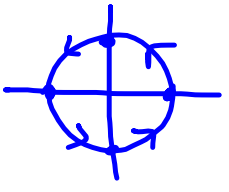
The functions describing the curve, C, traditionally use $f(t)$ to represent x and $g(t)$ to represent y. The independent variable t in this case is called a parameter .

The system of equations $\begin{cases} x = f(t) \\ y = g(t) \end{cases}$

is called a system of parametric equations. The parametrization of C endows it with an orientation and the arrows on C indicate the motion as values of t increase.

For example, this set of equations describes the unit circle, with the arrow indicating the orientation.

$$\begin{cases} x = \cos t \\ y = \sin t \end{cases} \qquad 0 \le t \le 2\pi$$

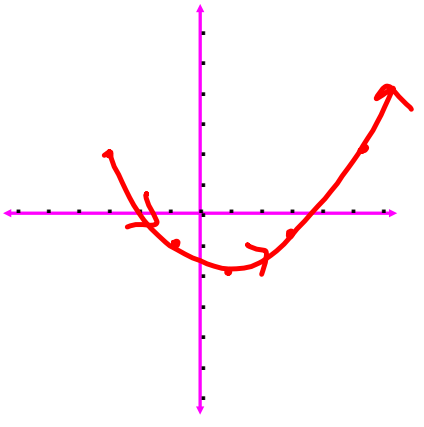


To sketch parametric equations, a chart is often useful.

Ex 1: Draw a chart for this set of equations and plot several points.

$$\begin{cases} x = 2t + 1 \\ y = t^2 - 2 \end{cases} \qquad t \ge -2$$

t	$x(t) = 2t + 1$	$y(t) = t^2 - 2$	$(x(t),y(t))$
-2	-3	2	(-3, 2)
-1	-1	-1	(-1, -1)
0	1	-2	(1, -2)
1	3	-1	(3, -1)
2	5	2	(5, 2)



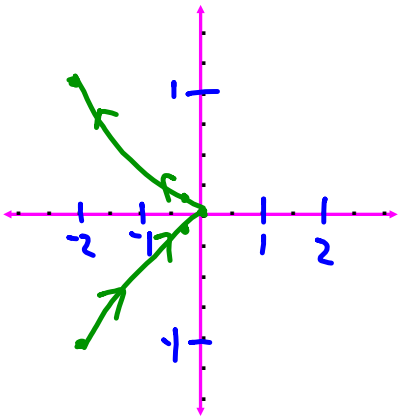
Ex 2: Plot this equation by following these steps.

$$\begin{cases} x = -2t^2 \\ y = t^3 \end{cases}$$ on the interval [-1,1]

a) Make a table of values.

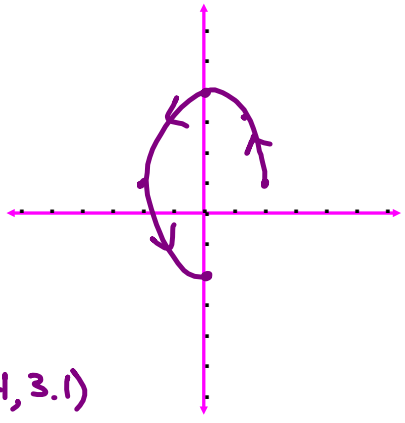
b) Plot the points, including orientation.

t	$x(t)$	$y(t)$	$(x(t),y(t))$
-1	-2	-1	$(-2,-1)$
$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{8}$	$(-\frac{1}{2},-\frac{1}{8})$
0	0	0	$(0,0)$
$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{8}$	$(-\frac{1}{2},\frac{1}{8})$
1	-2	1	$(-2,1)$



Ex 3: Plot this parametric curve with orientation.

$$\begin{cases} x = 2\cos t \\ y = 1 + 3\sin t \end{cases} \quad 0 \le t \le \frac{3\pi}{2}$$



t	$x(t)$	$y(t)$	$(x(t),y(t))$
0	2	1	$(2,1)$
$\frac{\pi}{4}$	$2\left(\frac{\sqrt{2}}{2}\right)=\sqrt{2}$	$1+\frac{3\sqrt{2}}{2}$	$\left(\sqrt{2},1+\frac{3\sqrt{2}}{2}\right)\simeq(1.4,3.1)$
$\frac{\pi}{2}$	0	4	$(0,4)$
π	-2	1	$(-2,1)$
$\frac{3\pi}{2}$	0	-2	$(0,-2)$