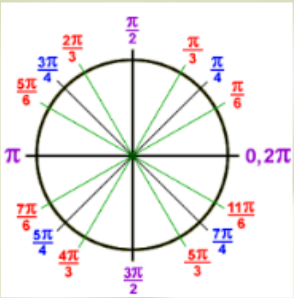
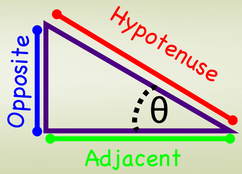
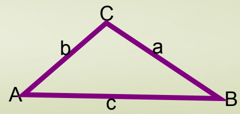
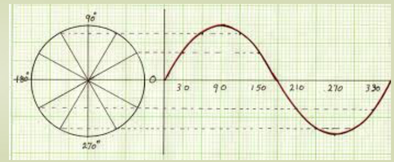








$$\sin^2 u + \cos^2 u = 1$$

$$\sin 2u = 2\sin u \cos u$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

Math 1060 ~ Trigonometry

25 Parametric Descriptions for Oriented Curves

Learning Objectives

In this section you will:

- **Eliminate the parameter in a pair of parametric equations.**
- **Parameterize curves given in Cartesian coordinates.**

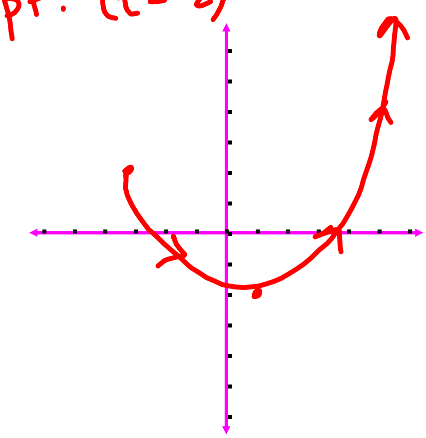
Eliminating the Parameter in Parametric Equations

To eliminate the parameter, we solve one equation for t and use substitution to arrive at a single equation in terms of x and y.

Ex 1: Eliminate the parameter in this system of equations from the previous lecture.

$$\begin{cases} x = 2t+1 & (1) \\ y = t^2 - 2 & (2) \end{cases} \quad t \geq -2$$

starting pt: $(t=-2)$
$(-3, 2)$



(1) $x - 1 = 2t$

$t = \frac{1}{2}x - \frac{1}{2}$

(2) $y = \left(\frac{1}{2}x - \frac{1}{2}\right)^2 - 2$

$y = \left(\frac{1}{2}(x-1)\right)^2 - 2$

$y = \left(\frac{1}{2}\right)^2 (x-1)^2 - 2$

$\boxed{y = \frac{1}{4}(x-1)^2 - 2}$

parabola w/ vertex $(1, -2)$

If the parametric equations have trigonometric expressions, using one of the Pythagorean Identities might be useful.

Ex 2: Eliminate the parameter in this set of equations and sketch the curve.

$$\begin{cases} x = 2\cos t & (1) \\ y = 1 + 3\sin t & (2) \end{cases} \quad 0 \le t \le \frac{3\pi}{2}$$

start pt: $(2, 1)$

end pt: $(0, -2)$

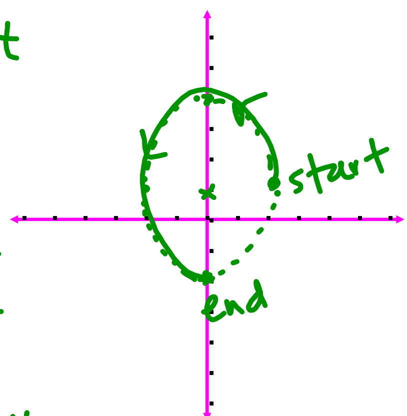
(1) $\frac{x}{2} = \cos t$

$\frac{x^2}{4} = \cos^2 t$

(2) $y - 1 = 3\sin t$

$\frac{y-1}{3} = \sin t$

$\left(\frac{y-1}{3}\right)^2 = \sin^2 t$



know: $\cos^2 t + \sin^2 t = 1$

$$\boxed{\frac{x^2}{4} + \frac{(y-1)^2}{9} = 1}$$ ellipse w/ center $(0,1)$

Ex 3: Find a parameterization for each of these curves and sketch each one.

a) $y = -2x+3$ from (2,-1) ① to (0,3) ②

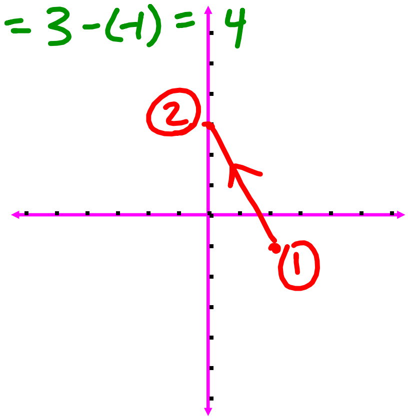
$\Delta x = 0-2 = -2$

$\Delta y = 3-(-1) = 4$

start w/:

$$\begin{cases} x = t \\ y = -2t+3 \end{cases}$$

from $t=2$ ①
to $t=0$ ②



OR

$$\begin{cases} x = 2 + -2t \\ y = -1 + 4t \end{cases} \quad t \in [0,1]$$

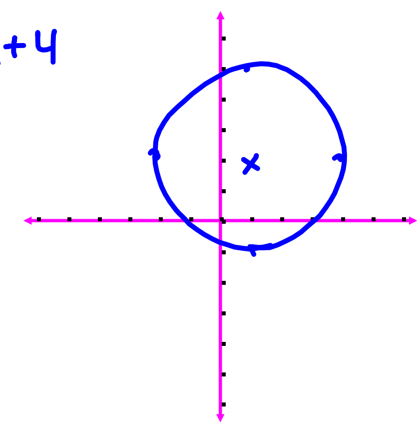
at $t=1$, $x=0$, $y=3$ ✓

b) $x^2 - 2x + y^2 - 4y = 4$

circle: center (1,2)
radius 3

$(x^2-2x+1)+(y^2-4y+4) = 4+1+4$

$(x-1)^2+(y-2)^2 = 9$



notice: $(3\sin\theta)^2 + (3\cos\theta)^2$

$= 9(\sin^2\theta + \cos^2\theta) = 9$

want:

$x-1 = 3\cos\theta$ and $y-2 = 3\sin\theta$

$x = 1+3\cos\theta$, $y = 2+3\sin\theta$, $\theta \in [0, 2\pi)$