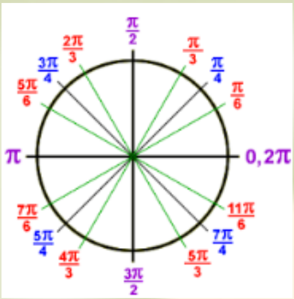
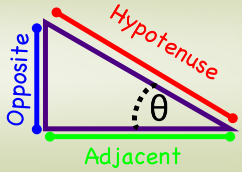
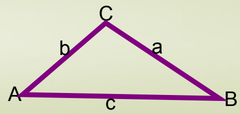
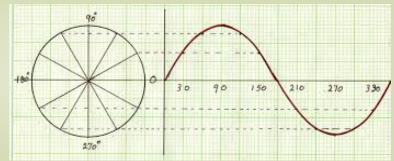








Math 1060 ~ Trigonometry

$\boxed{\sin^2 u + \cos^2 u = 1}$

$\boxed{\sin 2u = 2\sin u \cos u}$

$\boxed{\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}}$

$\boxed{c^2 = a^2 + b^2 - 2ab\cos C}$

5 Verifying Trigonometric Identities

Learning Objectives

In this section you will:

- **Learn and apply the Pythagorean identities and conjugates.**
- **Simplify trigonometric expressions.**
- **Prove that a trigonometric equation is an identity.**

The Pythagorean Identities:

1. $\boxed{\cos^2(\theta)+\sin^2(\theta)=1}$

   Common Alternate Forms: $1-\sin^2(\theta)=\cos^2(\theta)$ and $1-\cos^2(\theta)=\sin^2(\theta)$

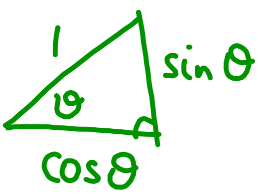
2. $1+\tan^2(\theta)=\sec^2(\theta)$, provided $\cos(\theta)\neq 0$

   Common Alternate Forms: $\sec^2(\theta)-\tan^2(\theta)=1$ and $\sec^2(\theta)-1=\tan^2(\theta)$

3. $1+\cot^2(\theta)=\csc^2(\theta)$, provided $\sin(\theta)\neq 0$

   Common Alternate Forms: $\csc^2(\theta)-\cot^2(\theta)=1$ and $\csc^2(\theta)-1=\cot^2(\theta)$

An identity is an eqn that's true for all values of the variable.



Note: to get (2):

$$\frac{\cos^2\theta}{\cos^2\theta}+\frac{\sin^2\theta}{\cos^2\theta}=\frac{1}{\cos^2\theta}$$

$$1+\tan^2\theta=\sec^2\theta$$

Ex 1: Given that sec $\theta = 2$, find tan θ, using the Pythagorean Identities.

① $1+\tan^2\theta=\sec^2\theta$

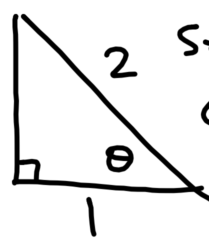
$1+\tan^2\theta=(2)^2$

$\tan^2\theta=3$

$\boxed{\tan\theta=\pm\sqrt{3}}$

OR

② $\sqrt{3}=d$



$\sec\theta=2$

$\Leftrightarrow \cos\theta=\frac{1}{2}$

$\tan\theta=\frac{\pm\sqrt{3}}{1}$

$d^2+1^2=2^2$

$d^2=3$

$d=\sqrt{3}$

$\boxed{\tan\theta=\pm\sqrt{3}}$

Ex 2: Verify the following identities. Assume that all quantities are defined.

(i.e. θ is in domain of original eqn)

a) $\tan\theta = (\sin\theta)(\sec\theta)$

$\sin\theta \sec\theta$

$= \sin\theta\left(\frac{1}{\cos\theta}\right)$

$= \frac{\sin\theta}{\cos\theta} = \tan\theta$

b) $\cot\theta = (\cos\theta)(\csc\theta)$

$\cos\theta \csc\theta$

$= \cos\theta\left(\frac{1}{\sin\theta}\right)$

$= \frac{\cos\theta}{\sin\theta} = \cot\theta$

c) $2(\sec\theta)(\tan\theta) = \dfrac{1}{1-\sin\theta} - \dfrac{1}{1+\sin\theta}$

$\frac{1}{1-\sin\theta} - \frac{1}{1+\sin\theta}$

$= \frac{1}{(1-\sin\theta)}\left(\frac{1+\sin\theta}{1+\sin\theta}\right) - \frac{1}{(1+\sin\theta)}\left(\frac{1-\sin\theta}{1-\sin\theta}\right)$

$= \frac{\cancel{1}+\sin\theta - (\cancel{1}-\sin\theta)}{1-\cancel{\sin\theta}+\cancel{\sin\theta}-\sin^2\theta}$

$= \frac{2\sin\theta}{1-\sin^2\theta} = \frac{2\sin\theta}{\cos^2\theta} = 2\left(\frac{\sin\theta}{\cos\theta}\right)\left(\frac{1}{\cos\theta}\right)$

$= 2\tan\theta\sec\theta$ ∎

$(a-b) \xrightarrow{\text{conjugate}} (a+b)$

Remember to use conjugates to simplify.

d) $\dfrac{1-\cos\theta}{\sin\theta} = \dfrac{\sin\theta}{1+\cos\theta}$

$\frac{1-\cos\theta}{\sin\theta} = \frac{(1-\cos\theta)}{\sin\theta}\left(\frac{1+\cos\theta}{1+\cos\theta}\right)$

$= \frac{1-\cos^2\theta}{\sin\theta(1+\cos\theta)}$

$= \frac{\sin^{\cancel{2}}\theta}{\cancel{\sin\theta}(1+\cos\theta)}$

$= \frac{\sin\theta}{1+\cos\theta}$ ∎

Strategies for Verifying Identities

Start by working with the expression on one side of the equation. Use legal mathematical steps on that expression until it is the expression on the other side of the equation.

Here are some suggestions of mathematical steps you can try.

- Try working on the more complicated side of the identity first.
- Use the Reciprocal and Quotient identities to write the expression in terms of only one or two of these: *sin x, cos x, tan x*, etc.
- Simplify complex fractions.
- Add rational expressions by obtaining common denominators.
- Use the Pythagorean identities to simplify sums and differences of squares.
- Multiply both numerator and denominator by a conjugate to take advantage of Pythagorean identities.
- If you get stuck working on one side, start over on the other side.