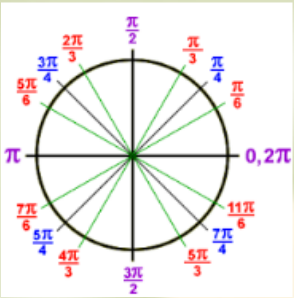
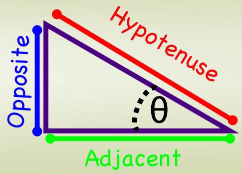
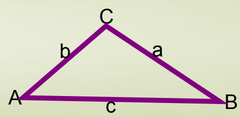
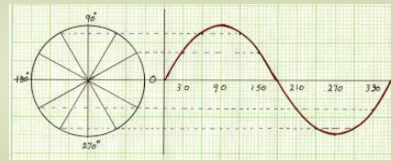








$$\sin^2 u + \cos^2 u = 1$$

$$\sin 2u = 2\sin u \cos u$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

Math 1060 ~ Trigonometry

6 Beyond The Unit Circle

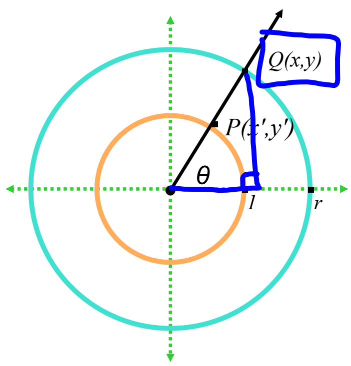
Learning Objectives

In this section you will:

- **Determine the values of the six trigonometric functions from the coordinates of a point on a circle, centered at the origin, with any radius *r*.**
- **Solve related application problems.**

Determining Sine and Cosine

Consider the acute angle θ drawn in standard position.

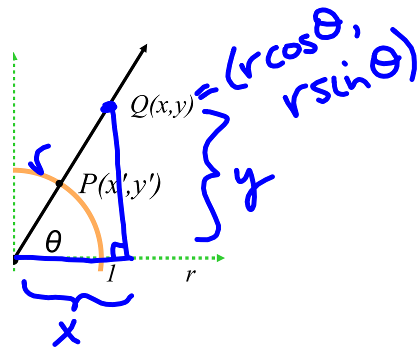


$\sin\theta = \frac{y}{r}$

$\Leftrightarrow \boxed{y = r\sin\theta}$

$\cos\theta = \frac{x}{r}$

$\Leftrightarrow \boxed{x = r\cos\theta}$



Q(x,y) is a point on the terminal side of θ which lies on the circle $x^2 + y^2 = r^2$.

P(x',y') is a point on the terminal side of θ which lies on the Unit Circle.

Theorem: If *Q(x,y)* is a point on the terminal side of an angle θ, plotted in standard position, which lies on the circle $x^2 + y^2 = r^2$, then

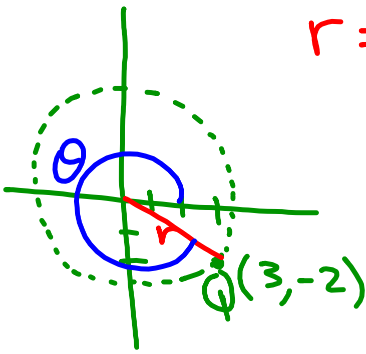
$\boxed{x = r\cos\theta \text{ and } y = r\sin\theta.}$

$$\cos\theta = \frac{x}{r} = \frac{x}{\sqrt{x^2+y^2}}$$

$$\sin\theta = \frac{y}{r} = \frac{y}{\sqrt{x^2+y^2}}$$

From these it is possible to determine all of the other four functions.

Ex 1: Determine the sine, secant and tangent of an angle which contains the point $Q(3, -2)$ when plotted in standard position.

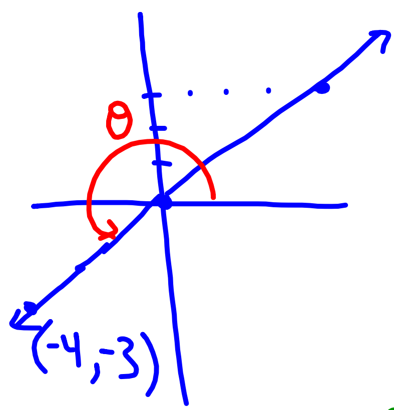


$$r = \sqrt{(3-0)^2 + (-2-0)^2}$$
$$= \sqrt{3^2 + 2^2}$$
$$= \sqrt{13}$$

$$\sin\theta = \frac{y}{r} = \frac{-2}{\sqrt{13}}$$
$$\cos\theta = \frac{x}{r} = \frac{3}{\sqrt{13}}$$
$$\sec\theta = \frac{\sqrt{13}}{3}$$
$$\tan\theta = \frac{y}{x} = \frac{-2}{3}$$

Ex 2: If the terminal side of θ lies on the line $3x - 4y = 0$ in the third quadrant, find the values of the six trigonometric functions of θ by finding a point on the line.

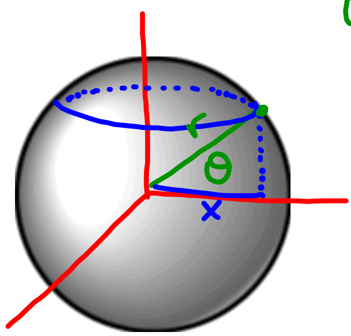
$$3x - 4y = 0$$
$$3x = 4y$$
$$y = \frac{3}{4}x$$



$$r = \sqrt{4^2 + 3^2}$$
$$= \sqrt{25}$$
$$= 5$$

$$\sin\theta = \frac{y}{r} = \frac{-3}{5}$$
$$\cos\theta = \frac{x}{r} = \frac{-4}{5}$$
$$\tan\theta = \frac{y}{x} = \frac{-3}{-4} = \frac{3}{4}$$
$$\csc\theta = \frac{-5}{3}$$
$$\sec\theta = \frac{-5}{4}$$
$$\cot\theta = \frac{4}{3}$$

Ex 3: Determine the radius of the circle of revolution for Salt Lake City, which is located at a latitude of 40.76º N. Assume the radius at the equator to be 3960 miles.



$\theta = 40.76°$

x = radius of SLC circle of revolution

we want to know $x = ?$

$r = 3960$ mi

$\cos\theta = \frac{x}{r} \iff x = r\cos\theta$

$x \approx 3960\cos(40.76°)$

$\boxed{x \approx 2999.5 \text{ mi}}$

WARNING: Be sure your calculator is in degree mode!