

Math 1060 ~ Trigonometry

7 Graphing The Cosine and Sine Functions

Learning Objectives

In this section you will:

- Graph the cosine and sine functions.
- Learn the properties of the cosine and sine functions, including domain and range, period, phase shift, amplitude and vertical shift.
- Identify cosine and sine functions as periodic functions.
- Determine whether a periodic function is even or odd.
- Use properties to graph periodic functions.
- Write an equation from the graph of a sine or cosine function.

$\sin^2 u + \cos^2 u = 1$

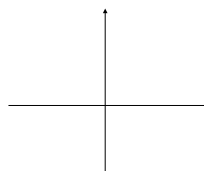
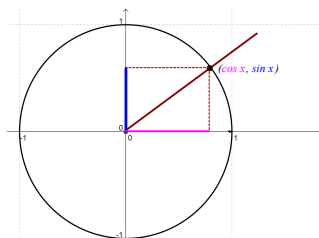
$\sin 2u = 2 \sin u \cos u$

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

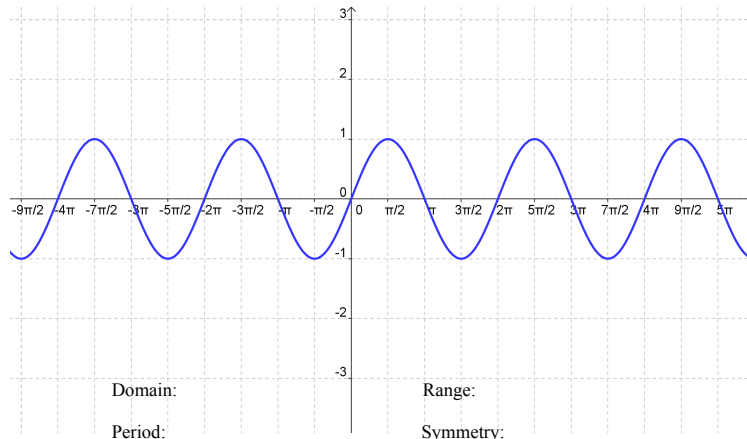
$c^2 = a^2 + b^2 - 2ab \cos C$

$$f(x) = \sin x$$

<http://tubs.geogebra.org/student/m45354?mobile=true>

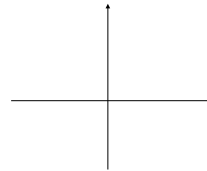
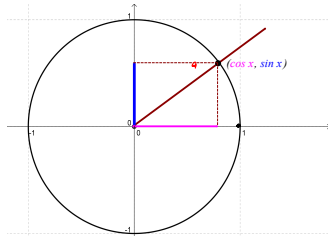


Graph of $f(x) = \sin x$

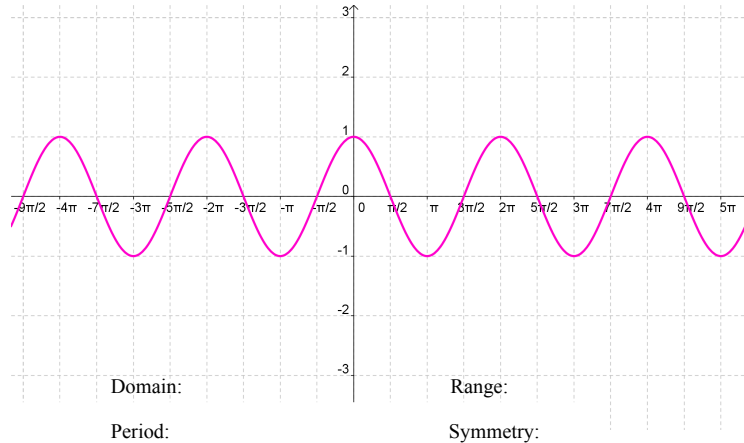


$f(x) = \cos x$

<http://tube.geogebra.org/student/m45354?mobile=true>



Graph of $f(x) = \cos x$

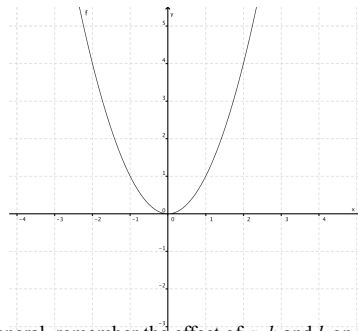


How can you graph $y = 2 \sin(x - \frac{\pi}{3}) + 1$?

This is a transformation of the basic $y = \sin x$ curve.

It may help to remember transformations to one of the algebraic functions.

How does the graph of $y = -3(x+2)^2 - 1$ relate to the graph of $y = x^2$?



In general, remember the effect of a , h and k on the graph of $y = x^2$.

$$y = a(x-h)^2 + k$$

$$y = A \sin(b(x-h)) + k$$

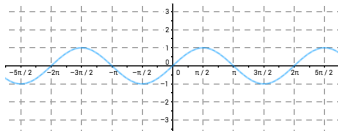
What effect do A , b , h and k have on the graph of trigonometric functions?

Let's look at it one part at a time: $y = A \sin x$

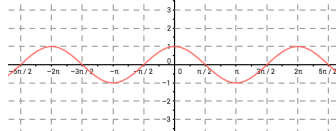
- Amplitude: $|A|$

Ex 1: Graph each of these.

$$y = 3 \sin x$$



$$y = -2 \cos x$$



Periodic Functions

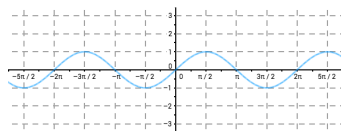
A function is periodic if there is a real number p so that $f(x+p) = f(x)$. The smallest positive number p , if it exists is called the period of f .

$$y = \sin(bx)$$

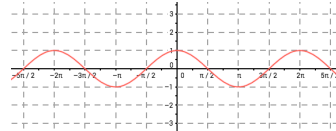
- Period =

Ex 2: Graph each of these.

$$y = \sin(2x)$$



$$y = \cos\left(\frac{1}{2}x\right)$$



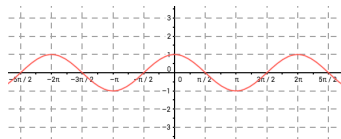
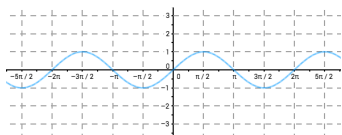
$$y = \sin(x-h)$$

- Horizontal shift (phase shift) =

Ex 3: Graph each of these.

$$y = \sin(x+\pi)$$

$$y = \cos\left(x - \frac{\pi}{2}\right)$$



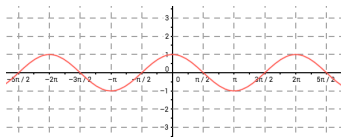
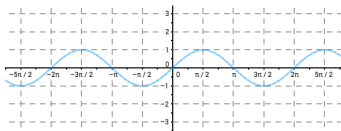
$$y = \sin(b(x - h))$$

- Period =
- Horizontal shift =

Ex 4: Graph each of these.

$$y = \sin(2x - \pi)$$

$$y = \cos\left(\left(\frac{1}{2}\right)x + \frac{\pi}{2}\right)$$

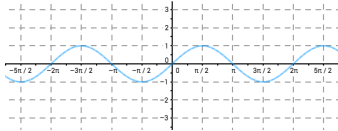


$$y = \sin(x) + k$$

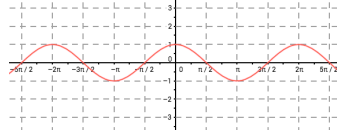
Vertical Shift :

Ex 5: Graph each of these.

$$y = \sin x - 2$$



$$y = \cos x + 1$$



So, when we graph a sine or cosine function there are these things to consider:

- Amplitude
- Period
- Phase shift (horizontal)
- Vertical shift

Ex 6: List the transformations of this function.

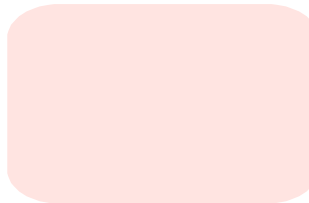
$$y = 3 \cos(2x - \pi) + 1$$

Amplitude

Period

Phase shift (horizontal)

Vertical shift



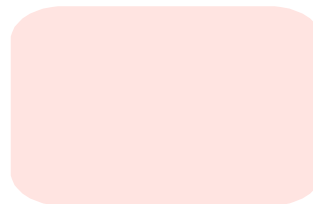
Ex 7: List the transformations of this function. $f(x) = -2\sin(4x - \pi) - 2$.

Amplitude

Period

Phase shift (horizontal)

Vertical shift



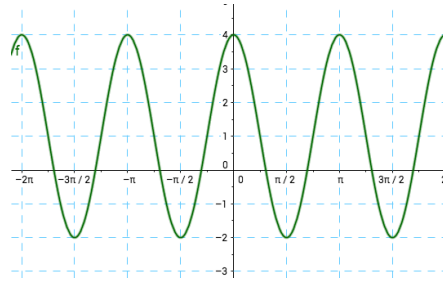
Ex 8: Analyze the transformations and write a function equation of this graph using the cosine function and then one using the sine function.

Period:

Amplitude:

Horizontal shift:

Vertical shift:



Here are some applets in case you want to play with the transformation variables.



[://www.analyzemath.com/trigonometry/sine.htm](http://www.analyzemath.com/trigonometry/sine.htm)

<http://tube.geogebra.org/student/m45354?mobile=true>

Here are instructions and the equation format from the text for graphing a periodic (sinusoidal) function.

For $\omega > 0$, the functions

$$C(x) = A \cos(\omega x + \phi) + B \text{ and } S(x) = A \sin(\omega x + \phi) + B$$

- have period $\frac{2\pi}{\omega}$
- have amplitude $|A|$
- have phase shift $-\frac{\phi}{\omega}$
- have vertical shift B