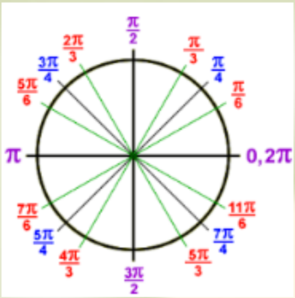
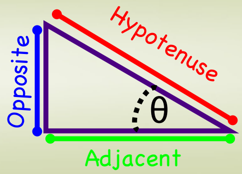
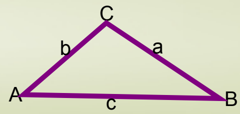
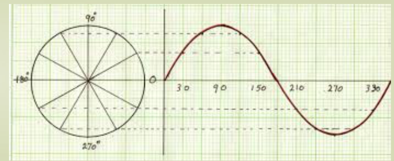








$\boxed{\sin^2 u + \cos^2 u = 1}$

$\boxed{\sin 2u = 2 \sin u \cos u}$

$\boxed{\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}}$

$\boxed{c^2 = a^2 + b^2 - 2ab\cos C}$

Math 1060 ~ Trigonometry

8 Graphing Other Trigonometric Functions

Learning Objectives

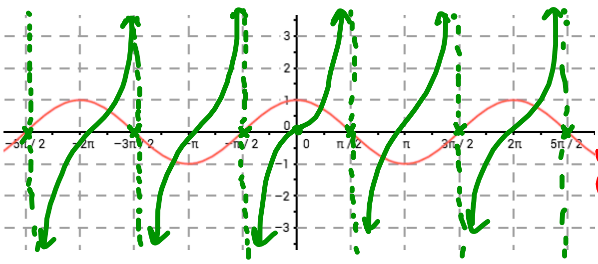
In this section you will:

- **Graph the tangent, cotangent, secant, and cosecant functions and their transformations. Identify the period and vertical asymptotes.**
- **Learn the properties of these functions, including domain and range; determine whether a function is even or odd.**
- **Recognize a function given the graph.**

note: $n \in \mathbb{Z}$ ← integers

↑ element of

$f(x) = \tan x = \dfrac{\sin x}{\cos x}$



y = tan x is an odd fn.

Vertical asymptotes:
where cos $x = 0$

$x = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \ldots$

Period:

π

Domain: $n \in \mathbb{Z}$

all $\mathbb{R}$ except $x = \frac{(2n+1)\pi}{2}$

Range:

all $\mathbb{R}$ (or $(-\infty, \infty)$)

Symmetry:

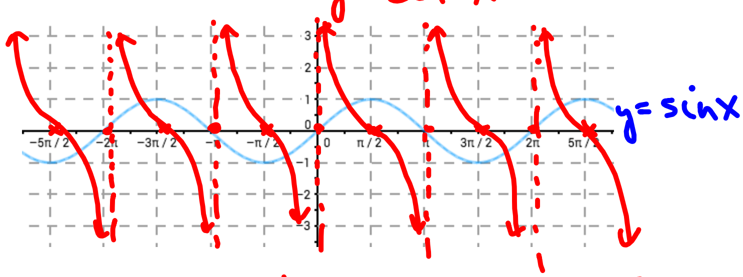
about origin

Increasing/decreasing:

increasing everywhere $y = \tan x$ is defined

$f(x) = \cot x = \dfrac{\cos x}{\sin x}$

y = cot x



y = cot x is an odd fn

Vertical asymptotes:
where $\sin x = 0$

$x = 0, \pm\pi, \pm 2\pi, \ldots$

Period: π

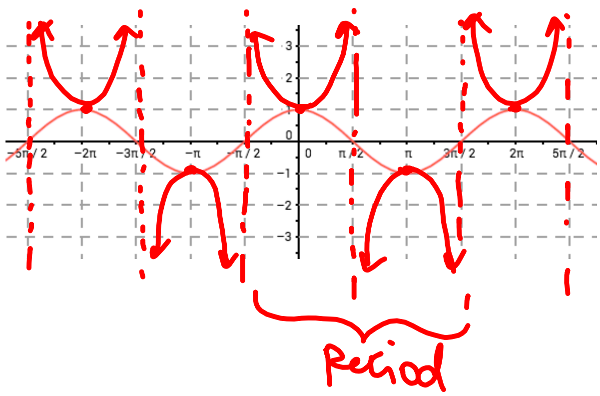
Domain: $x \in \mathbb{R}$, $x \neq n\pi$, $n \in \mathbb{Z}$

Range:
$y \in \mathbb{R}$ (OR $(-\infty, \infty)$)

Symmetry:
about origin

Increasing/decreasing:
decreasing wherever fn. is defined

$$f(x) = \sec x = \frac{1}{\cos x}$$



(x = odd multiple of $\pi/2$)

Vertical asymptotes:
where $\cos x = 0$

$x = \frac{(2n+1)\pi}{2}, n \in \mathbb{Z}$

Period:

2π

Domain: $x \in \mathbb{R}$, $x \neq \frac{(2n+1)\pi}{2}$ $n \in \mathbb{Z}$

Range:

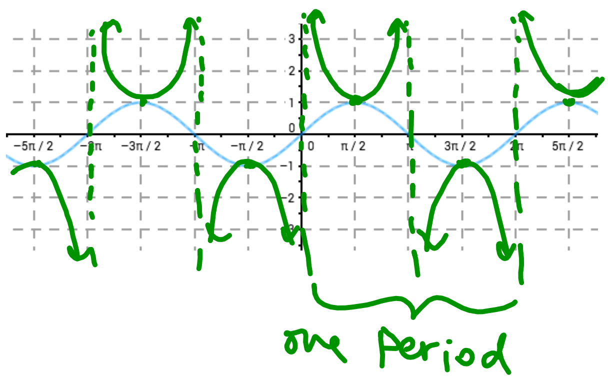
$(-\infty, -1] \cup [1, \infty)$

Symmetry:

across y-axis

$\Rightarrow y = \sec x$ is even fn.

$$f(x) = \csc x = \frac{1}{\sin x}$$



Vertical asymptotes:
where $\sin x = 0$

$x = n\pi, \ n \in \mathbb{Z}$

Period: 2π

Domain: $x \in \mathbb{R}$,
$x \neq n\pi, \ n \in \mathbb{Z}$

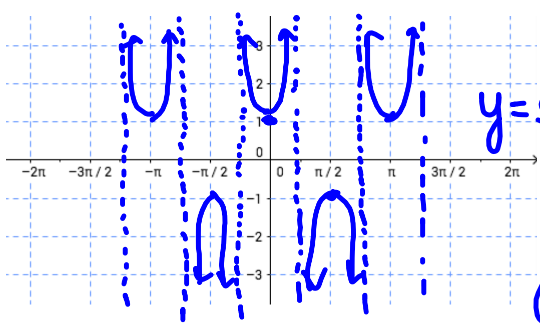
Range:
$(-\infty, -1] \cup [1, \infty)$

Symmetry:
about the origin
$\Rightarrow y = \csc x$ is odd fn.

Ex 1: List the transformations and sketch a graph.

$f(x) = \sec(2x) + 1$

impacts period and asymptotes



$y = \sec(2x)$

Period: $\frac{2\pi}{2} = \pi$

Asymptotes: $x =$ odd multiples of $\frac{\pi}{4}$

$x = \frac{(2n+1)\pi}{4}$

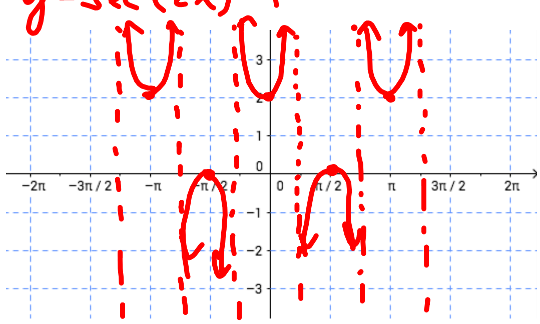
$n \in \mathbb{Z}$

(n is any integer)

Horizontal shift: none

Vertical shift: 1 (up)

$y = \sec(2x) + 1$



Ex 2: List the transformations for this function.

$$f(x) = \tan(2x - \frac{\pi}{2}) + 1$$

$2x - \frac{\pi}{2} = 0$

$2x = \frac{\pi}{2}$

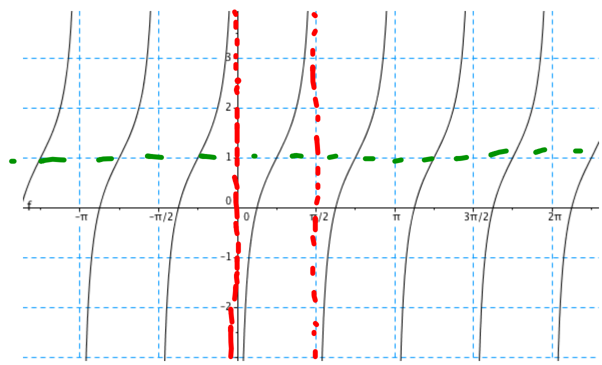
$x = \frac{\pi}{4}$

Period: $\frac{\pi}{2}$

Asymptotes: $x = \pm\frac{\pi}{2}\left(\frac{1}{2}\right) + \frac{\pi}{4} = \pm\frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2}, 0$ ("first" two VA)

Horizontal shift: $\frac{\pi}{4}$ (right)

Vertical shift: 1 (up)



VA: $x =$ any integer multiple of $\frac{\pi}{2}$

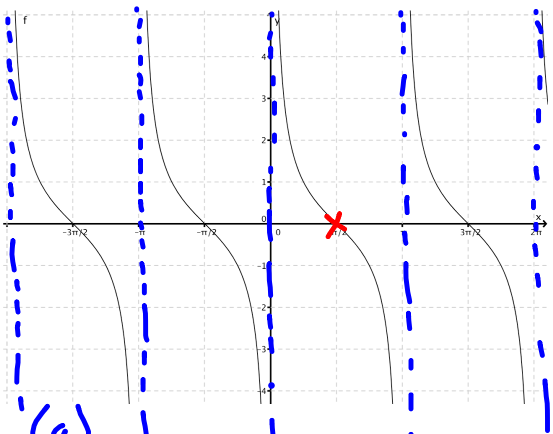
$x = \frac{n\pi}{2}, \ n \in \mathbb{Z}$

Ex 3: See if you can recognize which of the functions are represented in the graphs below.

a) Draw the asymptotes and write an equation for each of these graphs, assuming there are no transformations.

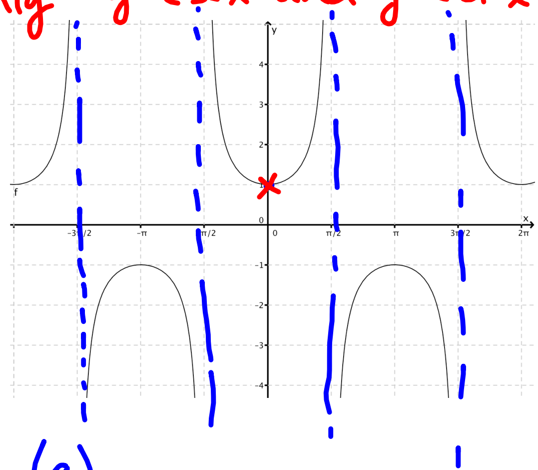
b) Write each function that is a co-function as a transformation of another function.

(only $y = \csc x$ and $y = \cot x$)

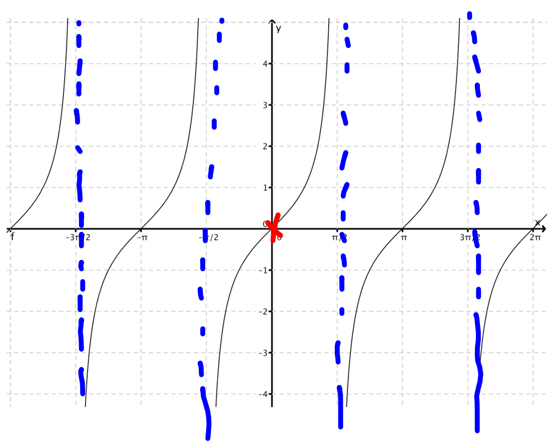


(a) $y = \cot x$

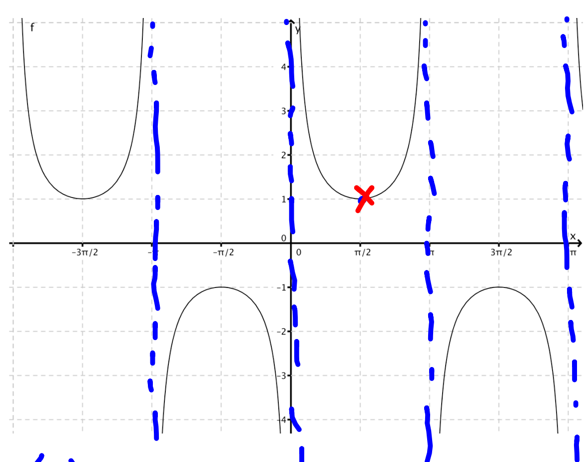
(b) $y = -\tan\left(x - \frac{\pi}{2}\right)$



(a) $y = \sec x$



(a) $y = \tan x$



(a) $y = \csc x$

(b) $y = \sec\left(x - \frac{\pi}{2}\right)$

(shift $\sec x = y$ curve to right by $\pi/2$)