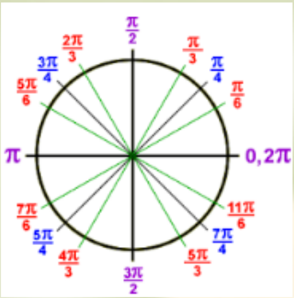
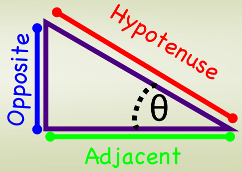
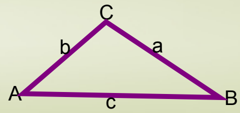
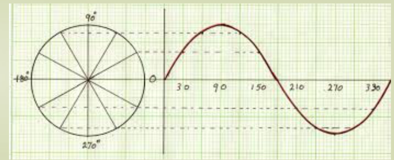








$\boxed{\sin^2 u + \cos^2 u = 1}$

$\boxed{\sin 2u = 2\sin u \cos u}$

$\boxed{\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}}$

$\boxed{c^2 = a^2 + b^2 - 2ab\cos C}$

Math 1060 ~ Trigonometry

9 Applications of Radian Measure

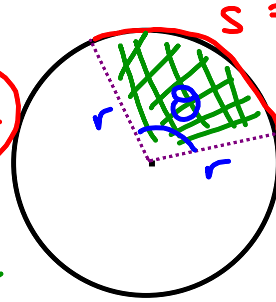
Learning Objectives

In this section you will:

- **Determine arc length.**
- **Determine area of a sector of a circle.**
- **Solve problems involving linear and angular velocity.**

Vocabulary

s = arc length



Arc: (length of) part of a circle

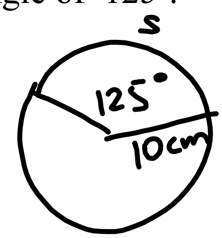
Sector: like a "pie piece"; portion of a circle bounded by 2 radii and intercepted arc

Length of a circular arc

$\boxed{s = r\theta}$

(note: θ must be in radians.)

Ex 1: Find the arc length along a circle of radius 10 cm subtended by an angle of 125°.



$s = r\theta$

$s = \cancel{10}^{5}\text{ cm}\left(\frac{25\pi}{\cancel{36}_{18}}\right)$

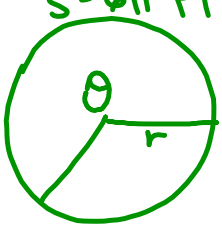
$s = \frac{125\pi}{18}\text{ cm} \simeq 21.8\text{ cm}$

$\theta = 125\cancel{°}\left(\frac{\pi}{180\cancel{°}}\right)$

$= \frac{25\pi}{36}$

Ex 2: What is the radius of a circle for which 2/3 of the circumference is 6π ft?

$s = 6\pi$ ft



$r = ?$

$s = r\theta$

$6\pi = r\left(\frac{4\pi}{3}\right)$

$\frac{3}{\cancel{4}_{2}\cancel{\pi}}\left(\cancel{6}^{3}\cancel{\pi}\right) = r$

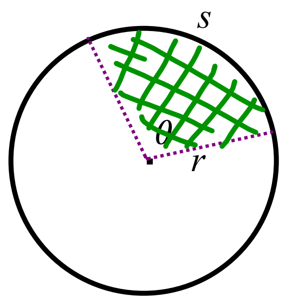
4.5 or $\frac{9}{2} = r$

$r = 4.5$ ft

$\theta = \frac{2}{3}(2\pi)$

$= \frac{4\pi}{3}$

Area of a Sector



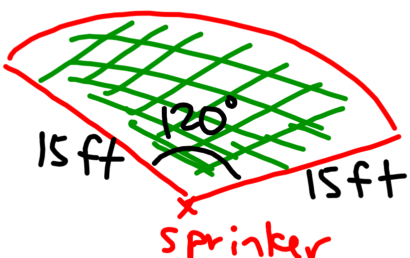
Area of whole circle $= \pi r^2$

$\Rightarrow$ area of sector $= \left(\dfrac{\theta}{2\cancel{\pi}}\right)\cancel{\pi} r^2 = \boxed{\tfrac{1}{2}\theta r^2}$

area of sector of circle w/ angle θ

Note: θ must be in radians!

Ex 3: A lawn sprinkler sprays a distance of 15 feet out and rotates back and forth at a 120º angle. What is the area that the sprinkler waters?



$A = \tfrac{1}{2}\theta r^2$

$A = \tfrac{1}{\cancel{2}}\left(\dfrac{\cancel{2}\pi}{3}\right)(15\text{ft})^2$

$A = \tfrac{\pi}{3}(225)\text{ ft}^2$

$\boxed{\begin{aligned} A &= 75\pi \text{ ft}^2 \\ &\approx 235.6 \text{ ft}^2 \end{aligned}}$

$\theta = 120^{\cancel{\circ}}\left(\dfrac{\pi}{180^{\cancel{\circ}}}\right)$

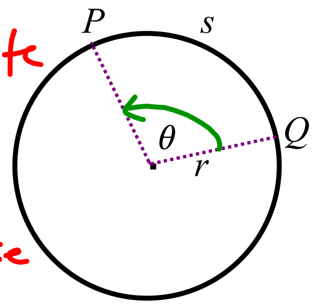
$= \dfrac{2\pi}{3}$

Linear and Angular Velocity

- velocity can be positive or negative (to indicate direction)

+ angular velocity : CC

− " " : clockwise



CC = counter clockwise

avg. Velocity $= \overline{v} = \dfrac{\text{displacement (distance)}}{\text{time}} = \dfrac{s}{t} = \dfrac{r\theta}{t} = r\left(\dfrac{\theta}{t}\right)$

read "v bar"

Average Angular Velocity $= \overline{\omega} = \dfrac{\text{change in angle}}{\text{time}} = \dfrac{\theta}{t}$

read "omega bar"

(units will be radians over time units)

Speed $= |\overline{v}|$ magnitude or absolute value of velocity; has no information about direction

<u>instantaneous speed</u> (for both linear and angular velocity) just means how fast it's going at one moment in time.

Velocity for Circular Motion

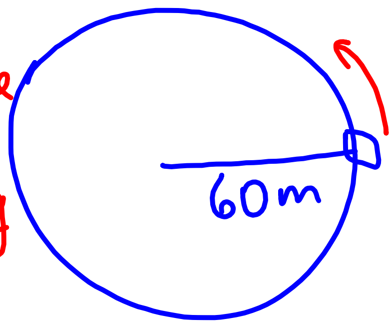
$$\boxed{v = r\,\omega}$$

from last page

$$V = r\left(\frac{\theta}{t}\right) = r\omega$$

Ex 4: The giant wheel in London, known as the Millennium Wheel has a radius of 60 meters. It completes one rotation in 30 minutes. What is the linear and angular velocity of a person riding in one of the cabins on the wheel? (It does not stop to pick up passengers, they hop on and off as it moves.)

assume it's rotating CC



(since it goes the same speed all the time, the instantaneous velocity = avg. velocity)

$V = r\omega$

① what is ω?

$$\omega = \frac{\theta}{t} = \frac{2\pi}{30\text{min}} = \frac{\pi}{15} \simeq 0.209 \text{ rad/min}$$

② what is v?

$$v = 60\text{m}\left(\frac{\pi}{15\text{min}}\right) = 4\pi \text{ m/min}$$

$$\simeq 12.6 \text{ m/min}$$