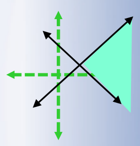
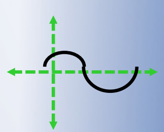
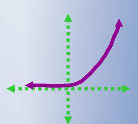
$5x-2y \leq 75$



$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$



$S = Pe^{rt}$



$APY = (1+\frac{r}{n})^n - 1$

Math 1090 ~ Business Algebra

Section 1.8 Graphical Linear Programming

Objectives:

- Maximize or minimize linear functions with constraints using graphical means.

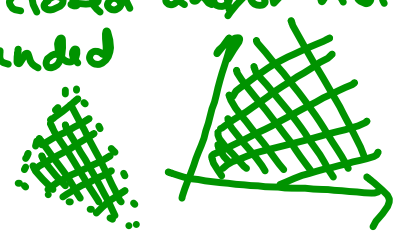
Graphical Linear Programming

Linear Programming : a technique to optimize a linear function given a set of linear constraints

Closed and Bounded Region

closed and bounded

examples of either not closed and/or not bounded



Constraints

linear inequalities whose solution set is graphed in 2-d

Feasible Region

the shaded region that's the solutn set for system of linear inequalities (constraints)

Optimal Solutions

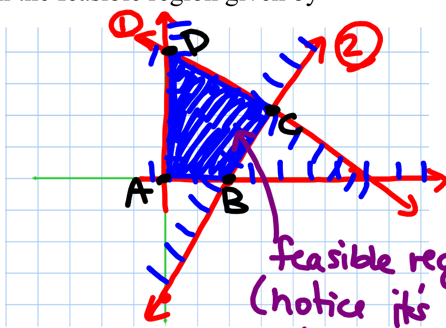
If we have a closed, bounded feasible region, the optimal solution for something measured on that feasible region is guaranteed to occur at a corner pt.

Graphical Linear Programming Procedures

1. Graph all constraints on the same axes.
2. Shade in the feasible region
3. Find and label vertices.
4. Plug in the corner points to the objective function to determine which gives the maximum or minimum as desired.

Ex 1: Find the minimum and maximum values of the objective function, $f = 4x + 3y$ on the feasible region given by

① $2x + 3y \le 12$
② $4x - 2y \le 8$
③ $x \ge 0, \quad y \ge 0$



test pt (0,0)
① $0 + 0 \le 12$ ✓
② $0 - 0 \le 8$ ✓

① (0, 4)
(6, 0)
② (0, -4)
(2, 0)

feasible region (notice it's closed & bounded)

⇒ we're guaranteed to find min and max at corner pts (for our objective fn)

vertices:

A (0, 0) D (0, 4)
B (2, 0) C (3, 2)

C is intersection of ① and ②

$-2(2x + 3y = 12) \qquad 4x - 2y = 8$

$$\begin{array}{lrl} ① & -4x - 6y &= -24 \\ ② & + \; 4x - 2y &= 8 \\ \hline & -8y &= -16 \\ & y &= 2 \end{array}$$

$$\begin{aligned} ② \quad 4x - 2(2) &= 8 \\ 4x - 4 &= 8 \\ 4x &= 12 \\ x &= 3 \end{aligned}$$

fn to optimize (objective fn):

$f = 4x + 3y$

A (0, 0) $\quad f = 4(0) + 3(0) = 0$
B (2, 0) $\quad f = 4(2) + 3(0) = 8$
C (3, 2) $\quad f = 4(3) + 3(2) = 18$
D (0, 4) $\quad f = 4(0) + 3(4) = 12$

min is 0 at (0, 0)
max is 18 at (3, 2)

Ex 2: Minimize $g = 22x - 17y$ subject to these constraints.

① $8x + 5y \ge 100$ $(0,20)$ $(\frac{25}{2}, 0)$

② $12x + 25y \ge 360$ $(0, 14.4)$ $(30, 0)$

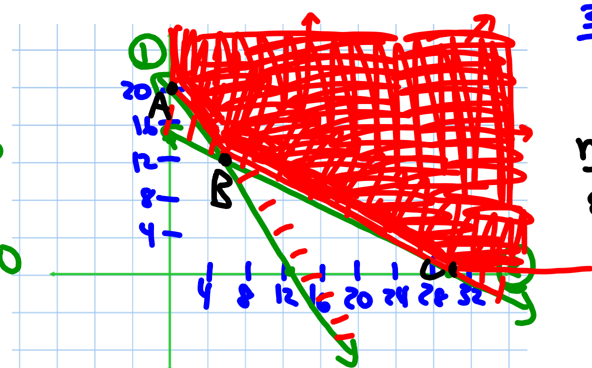
③ $x \ge 0, \quad y \ge 0$

$\frac{100}{8} = \frac{50}{4} = \frac{25}{2}$ or 12.5

test pt: $(0,0)$

① $0 \ge 100$ false

② $0 \ge 360$ false



$\frac{360}{25} = \frac{72}{5}$ or 14.4

not a closed & bounded region

⇒ we're not guaranteed to find a min/max but we might find min/max.

A (0,20)

B (5,12)

C (30,0)

B is intersectn pt of ① + ②

① $8x + 5y = 100$

② $12x + 25y = 360$

① $(5y = 100 - 8x)5$

$\Rightarrow 25y = 500 - 40x$

$\Rightarrow$ ② $12x + (500 - 40x) = 360$

$-28x + 500 = 360$

$-28x = -140$

$x = 5$

① $\Rightarrow 5y = 100 - 8(5)$

$5y = 60$ $\Rightarrow$ B is $(5, 12)$

$y = 12$

$g = 22x - 17y$

A $g = 22(0) - 17(20) = -340$

B $g = 22(5) - 17(12) = -94$

C $g = 22(30) - 17(0) = 660$

Conclusion:

NO min value of g since y can get outrageously big ⇒ $-17y$ gets smaller & smaller (i.e. closer & closer to $-\infty$)

Ex 3: A contractor builds two types of homes. The Carolina requires one lot, \$160,000 capital and 160 worker-days of labor. The Savannah requires one lot, \$240,000 capital and 160 worker-days of labor. The contractor owns 300 lots and has \$48,000,000 available capital and 43,200 worker-days of labor. The profit on the Carolina is \$40,000 and on the Savannah, it's \$50,000. How many of each type of home should be built to maximize profit? What is the maximum profit?

x = # of Carolina homes

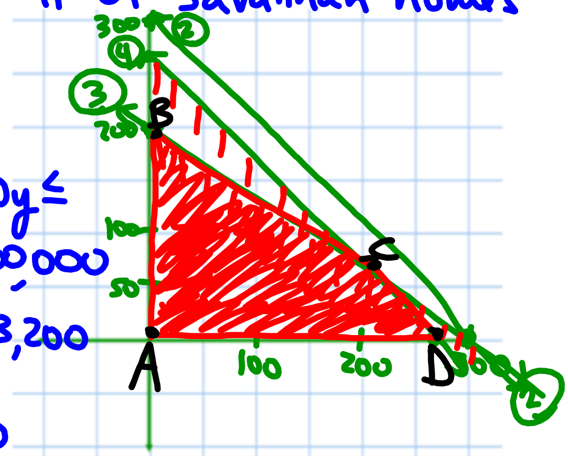
y = # of Savannah homes

① $x \geq 0,\ y \geq 0$

lots ② $x + y \leq 300$

\$ ③ $160{,}000x + 240{,}000y \leq 48{,}000{,}000$

labor ④ $160x + 160y \leq 43{,}200$



⇔ ③ $2x + 3y \leq 600$

⇔ ④ $x + y \leq 270$

int of ③ & ④:

③ $2x + 3y = 600$

④ $-2(x + y = 270)$

③ $2x + 3y = 600$

$+\ -2x - 2y = -540$

$y = 60$

④ $x + 60 = 270$

$x = 210$

A (0,0)

B (0,200) y-int. of ③

C (210, 60) int pt of ③ & ④

D (270,0) x-int. of ④

Profit = 40000x + 50000y

A (0,0) Profit = \$0

B (0,200) Profit = \$10,000,000

C (210,60) Profit = \$11,400,000

D (270,0) Profit = \$10,800,000

⇒ build 210 Carolina homes and 60 Savannah homes for max profit of \$11,400,000