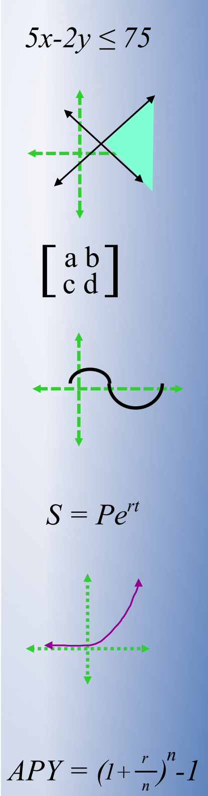

Math 1090 ~ Business Algebra

Section 2.3 Gauss-Jordan Elimination

Objectives:

- Set up an Augmented Matrix to represent a set of linear equations.
- Perform elementary row operations to a matrix.
- Manipulate the matrix to provide a solution to the set of linear equations.
- Recognize when there is more than one solution or none at all.

Vocabulary

Augmented Matrix: A matrix that represents a system of linear equations. ex $\begin{cases} 2x+y=3 \\ 5x-6y=7 \end{cases}$ augmented matrix: $\left[\begin{array}{cc:c} 2 & 1 & 3 \\ 5 & -6 & 7 \end{array}\right]$

Elementary Row Operations:

3 things we're allowed to do to aug. matrix

1. Switch two rows.

   ex $\left[\begin{array}{cc:c} 5 & -6 & 7 \\ 2 & 1 & 3 \end{array}\right]$

2. Multiply a row by a nonzero constant.

   ex $\left[\begin{array}{cc:c} 5 & -6 & 7 \\ 12 & 6 & 18 \end{array}\right]$

3. Replace one row with the result of adding it to a nonzero multiple of another row.

   ex $\left[\begin{array}{cc:c} 5 & -6 & 7 \\ 17 & 0 & 25 \end{array}\right]$

Gauss-Jordan Elimination: A process for solving a system of linear equations, using elementary row operations until we have a triangular matrix like this:

$$\left[\begin{array}{ccc:c} 1 & 3 & 4 & 5 \\ 0 & 1 & 2 & 7 \\ 0 & 0 & 1 & -4 \end{array}\right]$$

$$\begin{aligned} x + 3y + 4z &= 5 \\ y + 2z &= 7 \\ z &= -4 \end{aligned}$$

- have ones on the diagonal
- zeros below the diagonal

Ex 1: Solve. $3x - y = 3$

$x + z = 3$

$2x - y + z = 2$

$$\left[\begin{array}{ccc|c} 3 & -1 & 0 & 3 \\ 1 & 0 & 1 & 3 \\ 2 & -1 & 1 & 2 \end{array}\right]$$

$$\begin{array}{cc} & \begin{array}{cccc} -3 & 0 & -3 & -9 \end{array} \\ \begin{array}{c} (-3) \\ \\ \\ \end{array} & \left[\begin{array}{ccc|c} 1 & 0 & 1 & 3 \\ 3 & -1 & 0 & 3 \\ 2 & -1 & 1 & 2 \end{array}\right] \end{array}$$

$$\begin{array}{cc} & \begin{array}{cccc} -2 & 0 & -2 & -6 \end{array} \\ \begin{array}{c} (-2) \\ \\ \\ \end{array} & \left[\begin{array}{ccc|c} 1 & 0 & 1 & 3 \\ 0 & -1 & -3 & -6 \\ 2 & -1 & 1 & 2 \end{array}\right] \end{array}$$

$$\begin{array}{cc} \begin{array}{c} \\ (-1) \\ \\ \end{array} & \left[\begin{array}{ccc|c} 1 & 0 & 1 & 3 \\ 0 & -1 & -3 & -6 \\ 0 & -1 & -1 & -4 \end{array}\right] \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 3 \\ 0 & 1 & 3 & 6 \\ 0 & -1 & -1 & -4 \end{array}\right]$$

$$\begin{array}{cc} \begin{array}{c} \\ \\ (\frac{1}{2}) \end{array} & \left[\begin{array}{ccc|c} 1 & 0 & 1 & 3 \\ 0 & 1 & 3 & 6 \\ 0 & 0 & 2 & 2 \end{array}\right] \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 3 \\ 0 & 1 & 3 & 6 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

this is upper triangular form

① convert from system of linear eqns to an augmented matrix

② use elementary row ops to get it into upper triangular form

③ go back to linear eqns form from last matrix and finish solving!

① $x + z = 3$

② $y + 3z = 6$

③ $z = 1$ ✓

$\Rightarrow$ ② $y + 3(1) = 6$

$y = 3$ ✓

$\Rightarrow$ ① $x + 1 = 3$

$x = 2$ ✓

$\Rightarrow$ solution: $\boxed{(2,3,1)}$

Ex 2: Solve. $-2x+y=1$
$2x-y=7$

$$\left[\begin{array}{cc:c} -2 & 1 & 1 \\ 2 & -1 & 7 \end{array}\right]$$

$$\left[\begin{array}{cc:c} -2 & 1 & 1 \\ 0 & 0 & 8 \end{array}\right] \Rightarrow \begin{array}{c} -2x+y=1 \\ 0\neq 8 \end{array}$$

this can't be true

$\Rightarrow \boxed{N.S.}$

Ex 3: Solve. $\begin{aligned} 10x+y&=6 \\ 3x+y+2z&=3 \\ 2x-y-2z&=2 \end{aligned}$

$$\left[\begin{array}{ccc|c} 10 & 1 & 0 & 6 \\ 3 & 1 & 2 & 3 \\ 2 & -1 & -2 & 2 \end{array}\right] \quad (-1) \qquad \begin{array}{cccc} -2 & 1 & 2 & -2 \end{array}$$

$$\left[\begin{array}{ccc|c} 10 & 1 & 0 & 6 \\ 3 & 1 & 2 & 3 \\ 1 & 2 & 4 & 1 \end{array}\right]$$

$$(-3) \quad \begin{array}{cccc} -3 & -6 & -12 & -3 \end{array} \quad \left[\begin{array}{ccc|c} 1 & 2 & 4 & 1 \\ 3 & 1 & 2 & 3 \\ 10 & 1 & 0 & 6 \end{array}\right]$$

$$(-10) \quad \begin{array}{cccc} -10 & -20 & -40 & -10 \end{array} \quad \left[\begin{array}{ccc|c} 1 & 2 & 4 & 1 \\ 0 & -5 & -10 & 0 \\ 10 & 1 & 0 & 6 \end{array}\right]$$

$$\left(-\tfrac{1}{5}\right) \left[\begin{array}{ccc|c} 1 & 2 & 4 & 1 \\ 0 & -5 & -10 & 0 \\ 0 & -19 & -40 & -4 \end{array}\right]$$

$$(19) \quad \begin{array}{cccc} 0 & 19 & 38 & 0 \end{array} \quad \left[\begin{array}{ccc|c} 1 & 2 & 4 & 1 \\ 0 & 1 & 2 & 0 \\ 0 & -19 & -40 & -4 \end{array}\right]$$

$$\left(-\tfrac{1}{2}\right) \left[\begin{array}{ccc|c} 1 & 2 & 4 & 1 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & -2 & -4 \end{array}\right] \qquad \left[\begin{array}{ccc|c} 1 & 2 & 4 & 1 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

① $x+2y+4z=1$

② $y+2z=0$

③ $z=2$ $\Rightarrow$ ② $y+2(2)=0$

$y=-4$

$\Rightarrow$ ① $x+2(-4)+4(2)=1$

$x=1$

$\Rightarrow$ soln: $\boxed{(1,-4,2)}$

Ex 4: Solve. $\begin{aligned} 3x-2y-7z&=0 \\ x-y-z&=1 \\ -x+2y-3z&=-4 \end{aligned}$

$$\left[\begin{array}{ccc|c} 3 & -2 & -7 & 0 \\ 1 & -1 & -1 & 1 \\ -1 & 2 & -3 & -4 \end{array}\right]$$

$$\begin{array}{c} \begin{array}{cccc} -3 & 3 & 3 & -3 \end{array} \\ (-3)\left[\begin{array}{ccc|c} 1 & -1 & -1 & 1 \\ 3 & -2 & -7 & 0 \\ -1 & 2 & -3 & -4 \end{array}\right] \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & -1 & -1 & 1 \\ 0 & 1 & -4 & -3 \\ -1 & 2 & -3 & -4 \end{array}\right]$$

$$\begin{array}{c} \begin{array}{cccc} 0 & -1 & 4 & 3 \end{array} \\ \begin{array}{c} \\ (-1) \\ \\ \end{array}\left[\begin{array}{ccc|c} 1 & -1 & -1 & 1 \\ 0 & 1 & -4 & -3 \\ 0 & 1 & -4 & -3 \end{array}\right] \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & -1 & -1 & 1 \\ 0 & 1 & -4 & -3 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

① $x-y-z=1$

② $y-4z=-3$

③ $0=0$ (true)

(there are infinitely many solutions) represented by an entire line in 3-d

② $\boxed{y=4z-3}$

$\Rightarrow$ ① $x-(4z-3)-z=1$

$x-4z+3-z=1$

$x-5z+3=1$

$x-5z=-2$

$\boxed{x=5z-2}$

line of intersection:

$$\begin{cases} x=5z-2 \\ y=4z-3 \\ z=z \end{cases}$$

or $(5z-2,\ 4z-3,\ z)$

or $(5t-2,\ 4t-3,\ t)$

or $(5p-2,\ 4p-3,\ p)$

Ex 5: Solve.
$$\begin{aligned} x+y+z&=1 \\ x-y-z&=1 \\ -x+y-z&=1 \end{aligned}$$

$$\begin{array}{c} \begin{array}{cccc} -1 & -1 & -1 & -1 \end{array} \\ (-1)\left[\begin{array}{ccc:c} 1 & 1 & 1 & 1 \\ 1 & -1 & -1 & 1 \\ -1 & 1 & -1 & 1 \end{array}\right] \end{array} \quad \left[\begin{array}{ccc:c} 1 & 1 & 1 & 1 \\ 0 & -2 & -2 & 0 \\ -1 & 1 & -1 & 1 \end{array}\right]$$

$$\left(-\tfrac{1}{2}\right)\left[\begin{array}{ccc:c} 1 & 1 & 1 & 1 \\ 0 & -2 & -2 & 0 \\ 0 & 2 & 0 & 2 \end{array}\right] \quad \left[\begin{array}{ccc:c} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 2 & 0 & 2 \end{array}\right]$$

① $x+y+z=1$

② $y+z=0$

③ $2y=2 \iff y=1 \Rightarrow$ ② $1+z=0$, $z=-1$

$\Rightarrow$ ① $x+1+-1=1$, $x=1$ $\Rightarrow$ Soln: $\boxed{(1,1,-1)}$