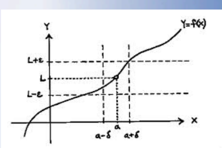
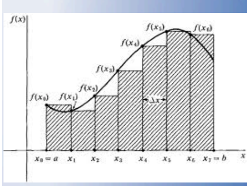


$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

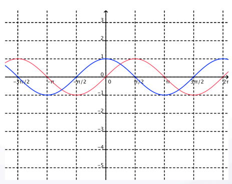
$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$

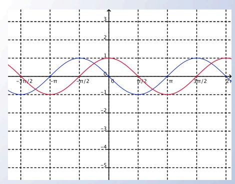


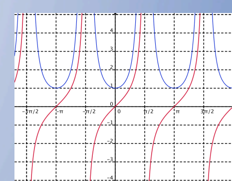
$$\lim_{max\,\Delta x_i \to 0} \sum_{1}^{n} f(x_i)\Delta x_i = \int_a^b f(x)dx$$

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

Derivatives of Trigonometric Functions

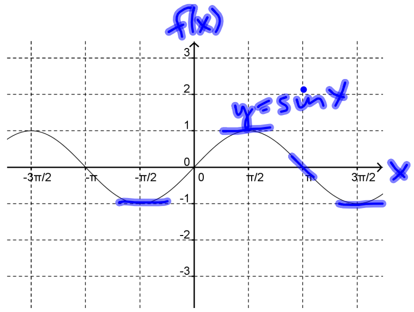


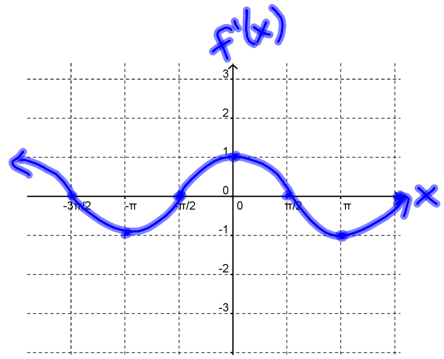




11B Derivatives Trig

The derivative of $f(x) = \sin x$





looks like $y = \cos x$

Use the definition of the derivative to find $D_x(\sin x)$.

$$D_x(\sin x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}, \quad f(x) = \sin x$$

$$= \lim_{h\to 0} \frac{\sin(x+h) - \sin x}{h}$$

$$= \lim_{h\to 0} \frac{\sin x \cos h + \cos x \sin h - \sin x}{h}$$

$$= \lim_{h\to 0} \frac{\sin x(\cos h - 1) + \cos x \sin h}{h}$$

$$= \lim_{h\to 0} \left[\frac{\sin x(\cos h - 1)}{h} + \cos x\left(\frac{\sin h}{h}\right)\right]$$

$$= \sin x\left(\lim_{h\to 0}\left(\frac{\cos h - 1}{h}\right)\right) + \cos x\left(\underbrace{\lim_{h\to 0}\frac{\sin h}{h}}_{1}\right)$$

$$= \sin x\left(\lim_{h\to 0} -\underbrace{\left(\frac{1-\cos h}{h}\right)}_{0}\right) + \cos x\,(1)$$

$$= 0 + \cos x$$

$$= \cos x$$

$$\boxed{D_x(\sin x) = \cos x}$$

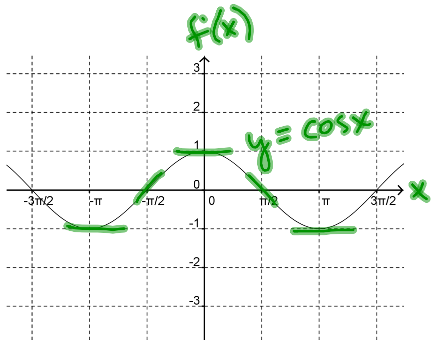
Note:

$$\lim_{\theta\to 0}\frac{\sin\theta}{\theta} = 1$$

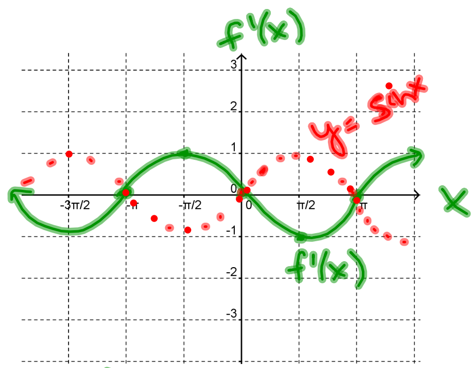
and

$$\lim_{\theta\to 0}\frac{1-\cos\theta}{\theta} = 0$$

The derivative of *f(x)* = *cos x*



derivative

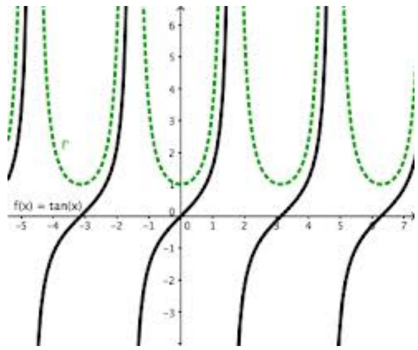


Is $D_x(\cos x) = \sin x$?

NO

but $\boxed{D_x(\cos x) = -\sin x}$

Here is a graph of y = tan x (black) and its derivative (green). Can you guess what its derivative might be?



green curve

$y = \sec^2 x$

$D_x(\sin x) = \cos x$

$D_x(\cos x) = -\sin x$

$D_x(\tan x) = \sec^2 x$

$D_x(\cot x) = -\csc^2 x$

$D_x(\csc x) = -\csc x \cot x$

$D_x(\sec x) = \sec x \tan x$

$$D_x(\tan x) = D_x\left(\frac{\sin x}{\cos x}\right)$$

$$= \frac{\cos x(\cos x) - \sin x(-\sin x)}{\cos^2 x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$$

$$= \frac{1}{\cos^2 x} = \sec^2 x$$

$$D_x(\cot x) = D_x\left(\frac{1}{\tan x}\right)$$

$$= \frac{\tan x(0) - 1(\sec^2 x)}{\tan^2 x}$$

$$= \frac{-\sec^2 x}{\tan^2 x}$$

$$= \frac{\frac{-1}{\cos^2 x}}{\frac{\sin^2 x}{\cos^2 x}} = \frac{-1}{\sin^2 x}$$

$$= -\csc^2 x$$

EX 1 Find y' for these functions.

a) $y = \sin^2 x = (\sin x)(\sin x)$

$y' = (\cos x)(\sin x) + \sin x(\cos x)$

$= 2\sin x\cos x$

b) $y = \cot x = \dfrac{\cos x}{\sin x}$

$y' = \dfrac{\sin x(-\sin x) - \cos x(\cos x)}{\sin^2 x}$

$y' = \dfrac{-\sin^2 x - \cos^2 x}{\sin^2 x}$

$y' = \dfrac{-(\sin^2 x + \cos^2 x)}{\sin^2 x}$

$= \dfrac{-1}{\sin^2 x} = -\csc^2 x$

c) $y = \dfrac{x\cos x + \sin x}{x^2 + 1}$

lo d-hi hi d-lo

$$y' = \frac{(x^2+1)\left(1\cdot\cos x + x(-\sin x) + \cos x\right) - (x\cos x + \sin x)(2x)}{(x^2+1)^2}$$

$$y' = \frac{(x^2+1)(2\cos x - x\sin x) - 2x^2\cos x - 2x\sin x}{(x^2+1)^2}$$

d) $y = \sin^2 x + \cos^2 x$

$y = 1$

$y' = 0$

EX 2 Find the equation of the tangent line to $y = \cot x$ at $x = \pi/4$

① need pt

② need slope

③ plug into $y - y_1 = m(x - x_1)$

① $(\pi/4,\ 1)$ $y = \cot(\pi/4)$

② $y' = -\csc^2 x$

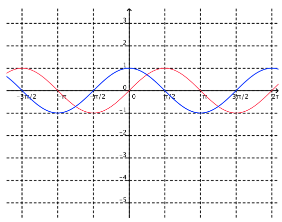
$m = -\csc^2(\pi/4)$

$= \dfrac{-1}{\sin^2(\pi/4)} = \dfrac{-1}{(1/\sqrt{2})^2} = \dfrac{-1}{1/2} = -2$

③ $y - 1 = -2(x - \pi/4)$

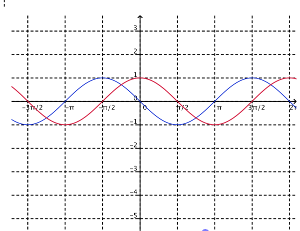
$y - 1 = -2x + \pi/2$

$\boxed{y = -2x + (\pi/2 + 1)}$



blue derivative curve
red original fn curve

$D_x(\sin x)$
$= \cos x$



$D_x(\cos x)$
$= -\sin x$

$D_x(\tan x) = \sec^2 x$