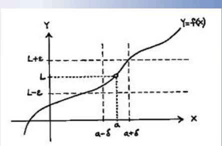
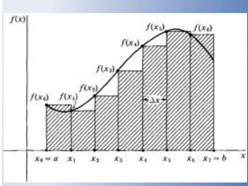


$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

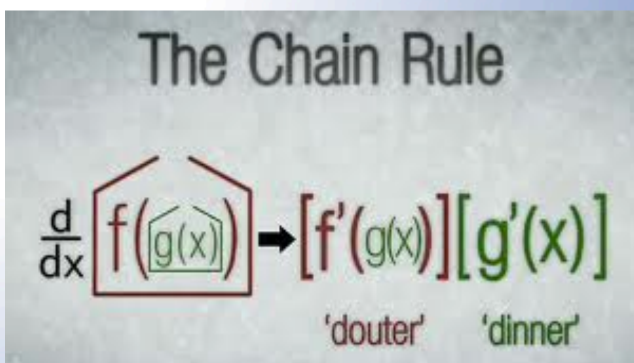
$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$



$$\lim_{\max \Delta x_i \to 0} \sum_{1}^{n} f(x_i)\Delta x_i = \int_a^b f(x)dx$$

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

The Chain Rule



The Chain Rule

$$D_x\left(f\left(g(x)\right)\right) = f'\left(g(x)\right)\left(g'(x)\right) \qquad \text{or} \qquad D_x y = (D_u y)(D_x u)$$

Basically, we differentiate from the 'outside-in.' This is very useful if we need to differentiate something like f(x) = $3(x^2-2x+1)^{80}$ and you really don't want to multiply it out.

EX 1 If $y=\left(3x^3-4x+5\right)^{10}$ find *y'*

✓ ① polynomial
✓ ② ^10

$y' = 10(3x^3-4x+5)^9(9x^2-4)$

EX 2 If $y=\dfrac{4}{\left(2x^7-6x^2\right)^5}$ find *y'*

① poly
✓ ② ^-5

$y = 4(2x^7-6x^2)^{-5}$

$y' = 4(-5)(2x^7-6x^2)^{-6}(14x^6-12x)$

$= \dfrac{-20(14x^6-12x)}{(2x^7-6x^2)^6}$

Ex 3 Find *f'(x)*:

a) $f(x) = \sin^2 x$ $= (\sin x)^2$

✓ ① $\sin$
✓ ② $\wedge 2$

$f'(x) = 2(\sin x)(\cos x)$

b) $f(x) = \sin(x^3)$

✓ ① $\wedge 3$
✓ ② $\sin$

$f'(x) = \cos(x^3)(3x^2)$

EX 3 (continued) Find *f′(x)*:

c) $f(x)=\left(\dfrac{2x+1}{x-5}\right)^4$

$\left(\text{note: } \dfrac{(2x+1)^4}{(x-5)^4}\right)$

$$f'(x)=4\left(\frac{2x+1}{x-5}\right)^3\left(\frac{(x-5)(2)-(2x+1)(1)}{(x-5)^2}\right)$$

d) $f(x)=\sin^2(4x)(2x^5-3)^3$ $= \underbrace{(\sin(4x))^2}_{①}\underbrace{(2x^5-3)^3}_{②}$

start w/ product rule

$$f'(x)=\overset{D①}{\big(2(\sin 4x)(\cos 4x)(4)\big)}\overset{②}{(2x^5-3)^3}$$

$$+\overset{①}{(\sin(4x))^2}\overset{D②}{\left[3(2x^5-3)^2(10x^4)\right]}$$

We can think of the chain rule as $\dfrac{dy}{dx} = \dfrac{dy}{du}\dfrac{du}{dx}$

EX 4 Find $\dfrac{dy}{dx}$

a) $y = \left[\underbrace{\left(2x^2+3\right)}_{①}\underbrace{cos(x)}_{②}\right]^4$

D① ② ① D②

$y' = \dfrac{dy}{dx} = 4\left((2x^2+3)\cos x\right)^3\left[(4x)\cos x + (2x^2+3)(-\sin x)\right]$

b) $y = \left(-3x + \dfrac{5}{x}\right)^{-4}$

$D_x\left(\dfrac{5}{x}\right) = D_x\left(5x^{-1}\right)$
$= -5x^{-2} = \dfrac{-5}{x^2}$

$\dfrac{dy}{dx} = -4\left(-3x + \dfrac{5}{x}\right)^{-5}\left(-3 + \dfrac{-5}{x^2}\right)$

12B Chain Rule