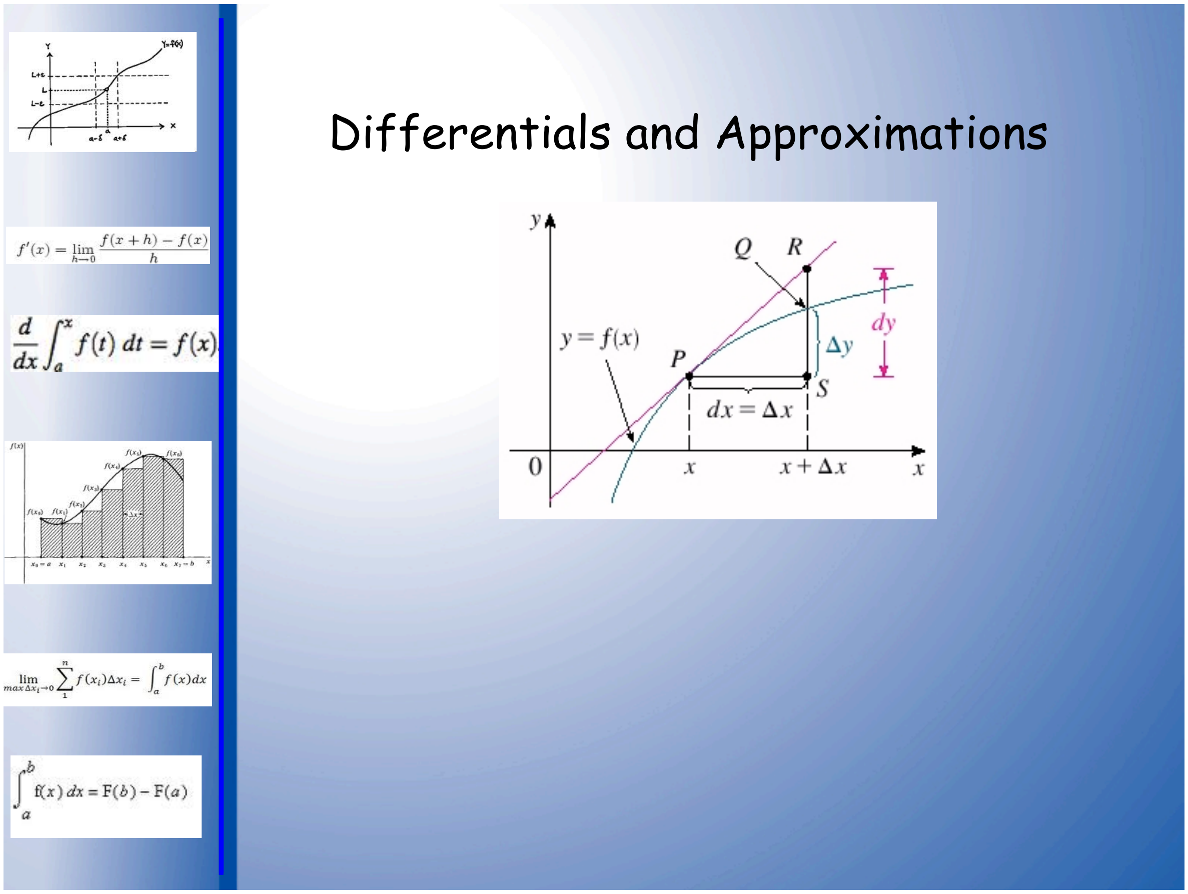
15.5 Differentials



<u>Differentials and Approximations</u>

We have seen the notation dy/dx and we've never separated the symbols.
Now, we'll give meaning to dy and dx as separate entities.

We know $\lim_{\Delta x \to 0} \frac{f(x_0+\Delta x)-f(x_0)}{\Delta x} = f'(x_0)$ gives the derivative (slope) of the function $f(x)$ at $x=x_0$.

If Δx is really small, then $\frac{f(x_0+\Delta x)-f(x_0)}{\Delta x} \approx f'(x_0)$

and $f(x_0+\Delta x)-f(x) \approx f'(x_0)\Delta x$

<u>Differentials</u>

Let $y=f(x)$ be a differentiable function of x. Δx is an arbitrary increment of x.

$dx = \Delta x$ (dx is called a differential of x.)
Δy is actual change in y as x goes from x to $x+\Delta x$.
i.e. $\Delta y = f(x+\Delta x)-f(x)$
$dy = f'(x)dx$ (dy is called the differential of y.)

15.5 Differentials

EX 1 Find dy.

a) $y = 4x^3-2x+5$

b) $y = 2\sqrt{x^4+6x}$

c) $y = \cos(x^3-5x+11)$

d) $y = \left(x^{10}+\sqrt{\sin(2x)}\right)^2$

Differentials can be used for approximations.

If $f(x+\Delta x)-f(x) \approx f'(x)\,\Delta x,$

then $f(x+\Delta x) \approx f(x) + f'(x)\,\Delta x.$

EX 2 Find a good approximation for $\sqrt{9.2}$ *without using a calculator.*

15.5 Differentials

EX 3 Use differentials to approximate the increase in the surface area of a soap bubble when its radius increases from 4 inches to 4.1 inches.

EX 4 The height of a cylinder is measured as 12 cm with a possible error of ± 0.1 cm. Evaluate the volume of the cylinder with radius 4 cm and give an estimate for the possible error in this value.