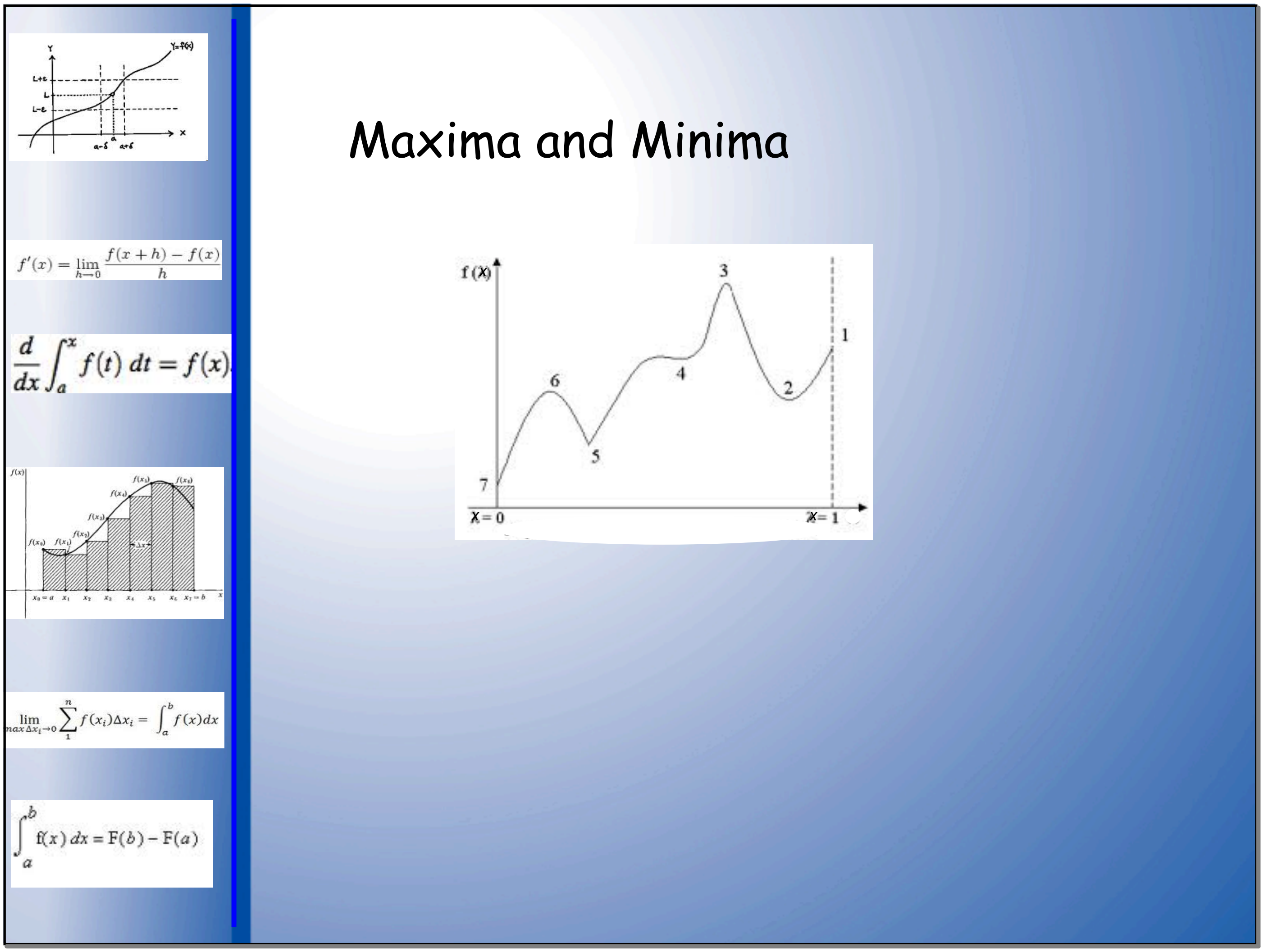

Maxima and Minima
f (X)
3
1
6
4
2
5
7
X = 0
X = 1
$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$
$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$
$\lim_{max\,\Delta x_i \to 0} \sum_1^n f(x_i)\Delta x_i = \int_a^b f(x)dx$
$\int_a^b f(x)\,dx = F(b) - F(a)$

Maxima and Minima

Definition: Let S, the domain of f, contain the point c.

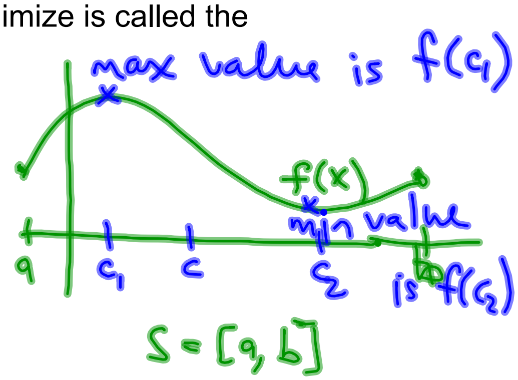
Then

i) $f(c)$ is a *maximum* value of f on S if $f(c) \geq f(x)$ for *all x in S.*

ii) $f(c)$ is a *minimum* value of f on S if $f(c) \leq f(x)$ for *all x in S.*

iii) $f(c)$ is an *extreme* value of f on S if it is the maximum or a minimum value.

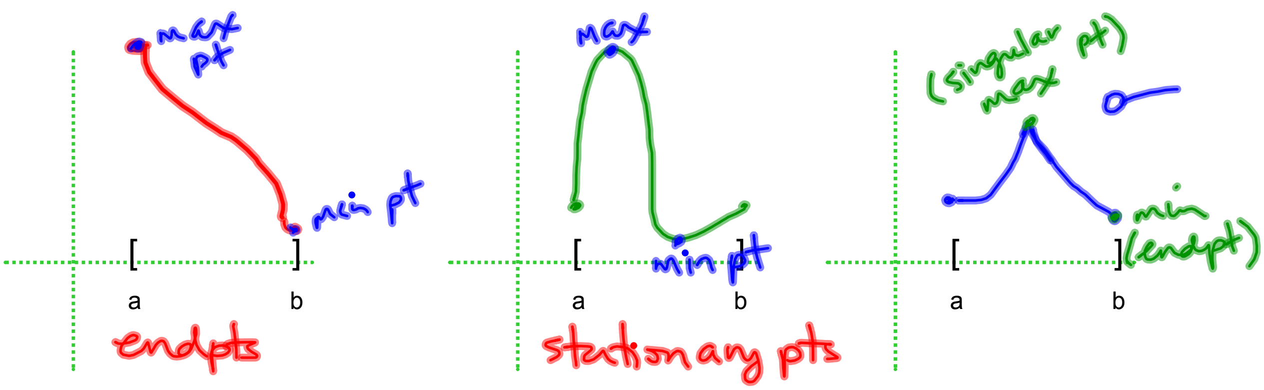
iv) the function we want to maximize or minimize is called the *objective function.*



16B Maxima Minima

Maximum - Minimum Existence Theorem

If f is ① continuous on a ② closed interval [a,b], then *f* attains both a maximum and minimum value on that interval.



These can occur in one of three ways:

1) endpoints of the closed interval.
2) stationary points where $f'(x) = 0$.
3) singular points where $f'(x)$ does not exist.

Note: We are guaranteed to find min & max pts on the curve if ① f(x) continuous and ② we're looking on a closed interval.

16B Maxima Minima

Ex 1 Find the minimum and maximum values of $f(x) = -2x^3 + 3x^2$ on [-1,3].

① continuous ② closed interval

$f(x) = -2x^3 + 3x^2$

$f'(x) = -6x^2 + 6x = 0$ (looking for stationary pts)

$6x(-x+1) = 0$

$x = 0, 1$

$f(0) = -2(0^3) + 3(0^2) = 0$

$f(1) = -2(1^3) + 3(1^2) = 1$

(0, 0)
(1, 1) } stationary pts

max (-1, 5)
min (3, -27) } endpts

* no singular pts

$f(-1) = -2(-1)^3 + 3(-1)^2 = 5$

$f(3) = -2(3^3) + 3(3^2) = -54 + 27 = -27$

Critical Point Theorem

Let *f* be defined on a closed interval, *I* containing the value *c*. If *f(c)* is an extreme value, then *c* is called a critical value.

(c, f(c)) is either

1) an endpoint of I or
2) a stationary point of *f*, i.e., $f'(c)=0$ *or*
3) a singular point of *f*, i.e., $f'(c)$ *DNE.*

main pt: critical pts are ① endpts, ② stationary pts or ③ singular pts.

16B Maxima Minima

EX 2 Find the minimum and maximum points for $f(x) = x^{2/5}$ on [-1,32]

① cont ② closed interval

$f'(x) = \frac{2}{5}x^{-3/5} = \frac{2}{5x^{3/5}} = 0?$

no $\Rightarrow$ no stationary pts.

but if $x=0$, the derivative DNE

$\Rightarrow$ there is singular pt at $x=0$, $\boxed{(0,0) \text{ min}}$

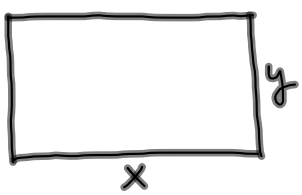
$f(0) = 0^{2/5} = 0$

endpts: $f(-1) = (-1)^{2/5} = ((-1)^2)^{1/5} = 1$ $(-1,1)$

$f(32) = 32^{2/5} = (2^5)^{2/5} = 2^2 = 4$ $\boxed{(32,4) \text{ max}}$

EX 3 Show that for a rectangle with perimeter of 30 inches, it has maximum area when it is a square.

goal: maximize area $\Rightarrow$ need Area fn



$30 = 2x + 2y$

$15 = x + y \rightarrow y = 15 - x$

$A = xy = x(15-x)$

$A = 15x - x^2$

endpts:

$0 \le x \le 15$

$A'(x) = 15 - 2x = 0$

$x = \frac{15}{2}$ (stationary pt x-value)

$(0,0)$ $A(0)=0$

$(15,0)$ $A(15)=0$

both min pts.

$\underbrace{\left(\frac{15}{2}, \frac{225}{4}\right)}_{\text{max pt}}$

$A\left(\frac{15}{2}\right) = 15\left(\frac{15}{2}\right) - \frac{15^2}{2^2}$

$= 15^2\left(\frac{1}{2} - \frac{1}{4}\right)$

$= \frac{15^2}{4} = \frac{225}{4} \text{ in}^2$

$\Rightarrow$ max area occurs when $x = \frac{15}{2}$

$\Rightarrow y = 15 - x = 15 - \frac{15}{2} = \frac{15}{2}$

$\Rightarrow$ this is square!

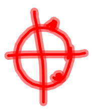
EX 4 Identify critical points (x,y) and specify the maximum and minimum values. (y-values)

$f(x) = x - 2\sin x$ on $[-2\pi, 2\pi]$.

① f is cont ② closed interval
⇒ we're guaranteed min & max

looking for: { endpts, stationary pts, singular pts }

$f'(x) = 1 - 2\cos x = 0$ (no singular pts)



$\frac{1}{2} = \cos x$

$x = \pi/3, -\pi/3, 5\pi/3, -5\pi/3$

⇒ min value $= \frac{-5\pi}{3} - \sqrt{3}$

max value $= 5\pi/3 + \sqrt{3}$

critical pts:

$(\pi/3, \frac{\pi}{3} - \sqrt{3})$ $(-2\pi, -2\pi)$

$(-\pi/3, \frac{-\pi}{3} + \sqrt{3})$ $(2\pi, 2\pi)$

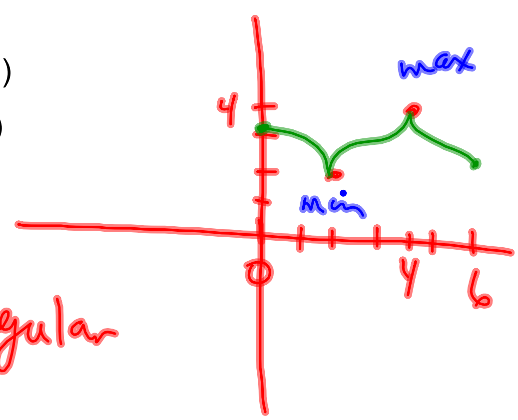
$(5\pi/3, \frac{5\pi}{3} + \sqrt{3})$

$(-5\pi/3, \frac{-5\pi}{3} - \sqrt{3})$

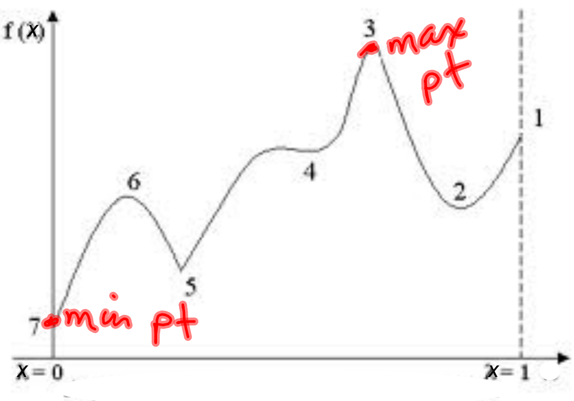
EX 5 Sketch the graph of a function with all of these characteristics:

1) continuous, but not necessarily differentiable.
2) has domain [0,6] ✓
3) reaches a maximum value of 4 (at x=4) ✓
4) reaches a minimum value of 2 (at x=2) ✓
5) has no stationary points.

(this means, min & max pts are singular pts)



16B Maxima Minima



note: this is half-open interval

- fn is cont.

EX 6 Find all inflection points for $f(x) = 2x^{\frac{1}{3}} - 1$.