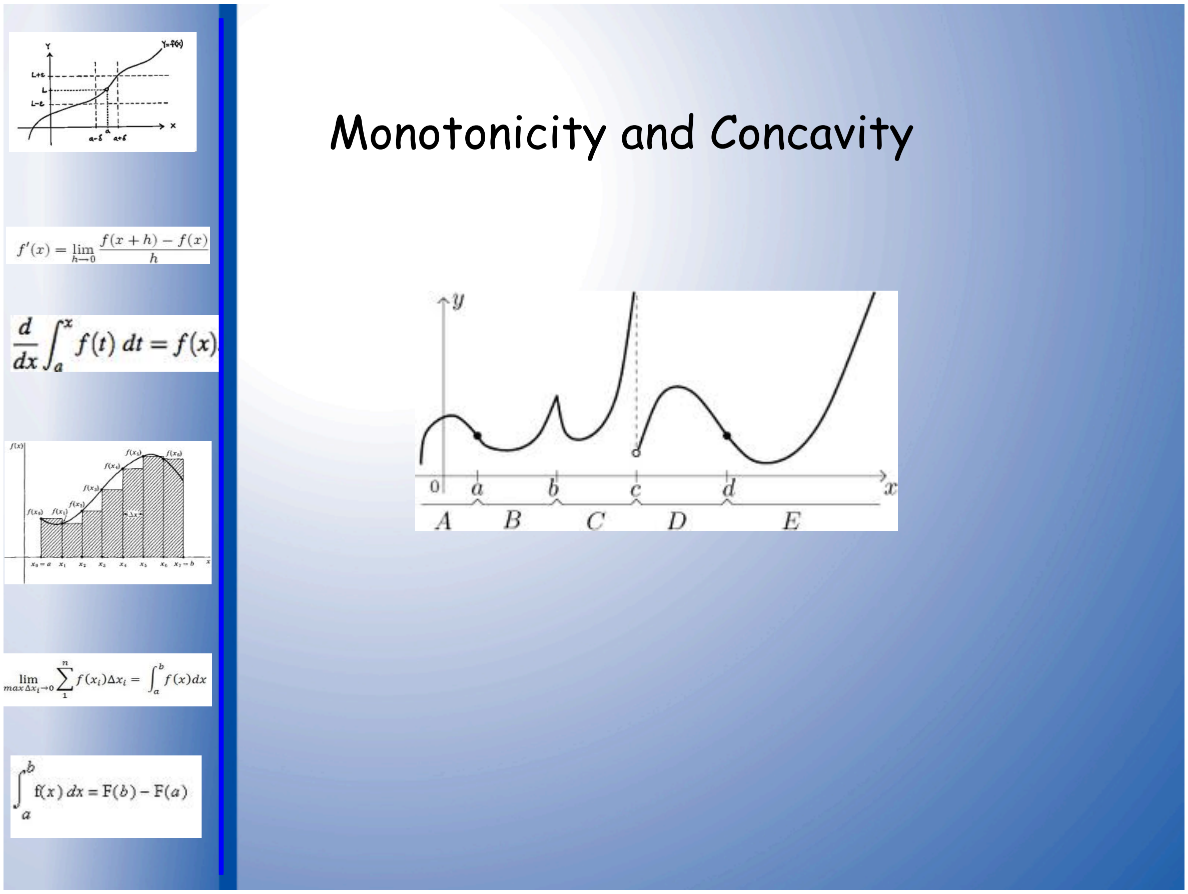
17 Monotonicity Concavity



<u>Definition</u>

Let *f* be defined on an interval I, (open, closed or neither), we say that:

1) *f* is <u>*increasing*</u> on I if for every x_1, x_2 in I $x_1 < x_2$ implies $f(x_1) < f(x_2)$.
2) *f* is <u>*decreasing*</u> on I if for every x_1, x_2 in I $x_1 < x_2$ implies $f(x_1) > f(x_2)$.
3) *f* is <u>strictly monotonic</u> on I if it is either increasing or decreasing on I .

<u>**Monotonicity Theorem**</u>

Let *f* be continuous on the interval, I and differentiable everywhere inside I .

1) if $f'(x) > 0$ for all x on the interval, then f is increasing on that interval.
2) if $f'(x) < 0$ for all x on the interval, then f is decreasing on that interval.

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EX 1 For each function, determine where *f* is increasing and decreasing.

a) $f(x) = x^3 + 3x^2 - 12$

b) $f(x) = \frac{x-1}{x^2}$

EX 2 Where is $f(x) = cos^2 x$ increasing and decreasing on the interval [**0**,**2**π]?

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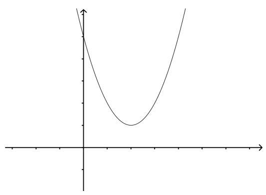
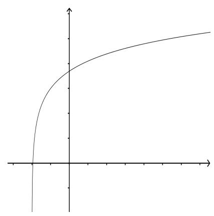
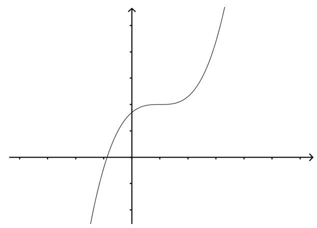
Definition

Let f be differentiable on an open interval, I .

f is concave up on I if f'(x) is increasing on I , and

f is concave down on I if f'(x) is decreasing on I .

Concavity Theorem

Let *f* be twice differentiable on an open interval, I.

If $f''(x) > 0$ for all x on the interval, then *f* is concave up on the interval.

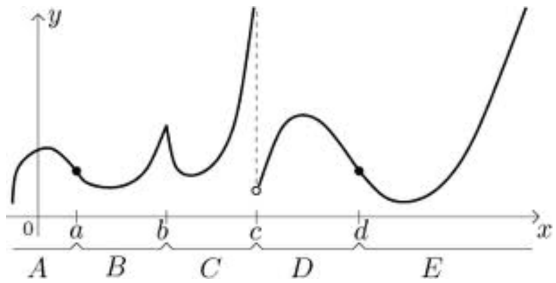
If $f''(x) < 0$ for all x on the interval, then *f* is concave down on the interval.

EX 3 Determine where this function is increasing, decreasing, concave up and concave down.

$f(x) = 4x^3 - 3x^2 - 6x + 12$

Inflection Point

Let *f* be continuous at *c*. We call *(c, f(c))* an inflection point of *f* if *f* is concave up on one side of *c* and concave down on the other side of *c*.



Inflection points will occur at *x*-values for which $f''(x) = 0$ or $f''(x)$ is undefined.

EX 4 For this function, determine where it is increasing and decreasing, where it is concave up and down, find all max/min and inflection points. Use this information to sketch the graph.

$f(x) = 8x^{1/3} - x^{4/3}$