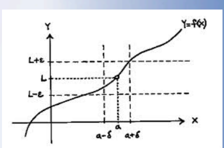
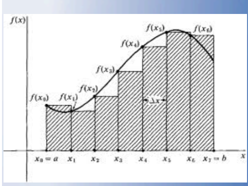


$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\frac{d}{dx} \int_a^x f(t)\, dt = f(x)$$



$$\lim_{max\,\Delta x_i \to 0} \sum_{1}^{n} f(x_i)\Delta x_i = \int_a^b f(x)dx$$

$$\int_a^b f(x)\, dx = F(b) - F(a)$$

Antiderivatives

Function $f(x)$	Antiderivative $F(x)$
1	x
$2x$	x^2
x^3	$\frac{1}{4}x^4$
$\cos x$	$\sin x$
$\sin 2x$	$-\frac{1}{2}\cos 2x$

Definition: Antiderivative

We call F an antiderivative of f on the interval, I, if

$D_x F(x)=f(x)$ on I.

ie. If $F'(x)=f(x)$ for all x on the interval.

(antiderivative "undoes" the derivative, up to a constant)

$$\boxed{\text{Power Rule Theorem: For every real value of } r \text{ except } r=-1, \text{ then } \int x^r\,dx = \frac{x^{r+1}}{r+1} + C}$$

integral sign

arbitrary constant

$\int$ dx, come together

remember: power rule for derivative

$D_x(x^r) = rx^{r-1}$

note: $D_x\left(\frac{x^{r+1}}{r+1}\right) = \frac{1}{r+1}(r+1)x^r = x^r$

Indefinite Integral is a linear operator.

(note: Indefinite Integral synonymous w/ antiderivative)

linear operator:

1. $\int (f(x)+g(x))\,dx = \int f(x)\,dx + \int g(x)\,dx$

(integration distributes through addition)

2. $\int k f(x)\,dx = k\int f(x)\,dx$, k is fixed constant

(integration commute w/ mult. by a constant)

EX 1 Evaluate the following integrals.

a) $\int\left(2x^4+3x^2-7\right)dx$ $= \int 2x^4\,dx + \int 3x^2\,dx - \int 7\,dx$

$$= 2\underbrace{\int x^4\,dx}_{\text{power rule}} + 3\underbrace{\int x^2\,dx}_{\text{power rule}} - 7\underbrace{\int 1\,dx}_{\text{power rule}}$$

$$= 2\left(\frac{x^5}{5}\right) + 3\left(\frac{x^3}{3}\right) - 7\left(\frac{x^1}{1}\right) + C$$

$$= \boxed{\frac{2}{5}x^5 + x^3 - 7x + C}$$

b) $\int\left(u^3-u^9\right)du$

$$= \frac{u^4}{4} - \frac{u^{10}}{10} + C$$

EX 2 Evaluate the following integrals.

a) $\int \left(\frac{1}{y^2} + y^{\frac{1}{3}} \right) dy$ $= \int \left(y^{-2} + y^{1/3} \right) dy$

$= \frac{y^{-1}}{-1} + \frac{y^{4/3}}{4/3} + c = \frac{-1}{y} + \frac{3}{4} y^{4/3} + c$

b) $\int \left(x^{-4} + \sqrt[3]{x^2} - \frac{3}{x^5} \right) dx$

$= \int \left(x^{-4} + x^{2/3} - 3x^{-5} \right) dx$

$= \frac{x^{-3}}{-3} + \frac{x^{5/3}}{5/3} - 3\left(\frac{x^{-4}}{-4} \right) + c$

$= \frac{-1}{3x^3} + \frac{3}{5} x^{5/3} + \frac{3}{4x^4} + c$

Theorem

$$\int \sin x \, dx = -\cos x + C$$

$$\int \cos x \, dx = \sin x + C$$

$\Longleftrightarrow$

$D_x(-\cos x) = \sin x$

$D_x(\sin x) = \cos x$

EX 3 $\int \left(t^2 - 2cost\right) dt$

$= \int t^2 dt - 2\int \cos t \, dt$

$= \frac{t^3}{3} - 2(\sin t) + c$

Generalized Theorem

(generalized power rule)

Let g be differentiable and r a rational number, $r \neq -1$, then

$$\int [g(x)]^r g'(x)\,dx = \frac{[g(x)]^{r+1}}{r+1} + C$$

(u-substitution)

EX 4 $\int \underbrace{(4x^3+1)}_{u}^4 \underbrace{12x^2\,dx}_{du} = \int u^4\,du$

let $u = 4x^3+1$

$\frac{du}{dx} = 12x^2$

$du = 12x^2\,dx$

$= \frac{u^5}{5} + c$

$= \boxed{\frac{(4x^3+1)^5}{5} + C}$

EX 5 $\int \underline{(5x^2+1)}\sqrt{\underbrace{5x^3+3x-2}}\,\underline{dx}$ $= \int \sqrt{u}\left(\frac{1}{3}\right)du$

$u = 5x^3+3x-2$

$\frac{du}{dx} = 15x^2+3 = 3(5x^2+1)$

$du = 3(5x^2+1)\,dx$

$\frac{1}{3}du = (5x^2+1)\,dx$

$= \frac{1}{3}\int u^{1/2}\,du$

$= \frac{1}{3}\left(\frac{u^{3/2}}{3/2}\right)+c$

$= \frac{1}{3}\left(\frac{2}{3}\right)(5x^3+3x-2)^{3/2}+c$

$= \boxed{\frac{2}{9}(5x^3+3x-2)^{3/2}+c}$

EX 6 $\int \frac{3y}{\sqrt{2y^2+5}}dy$ $= \int \frac{3\left(\frac{1}{4}\right)du}{\sqrt{u}}$

$u = 2y^2+5$

$\frac{du}{dy} = 4y \Rightarrow du = 4y\,dy$

$\frac{1}{4}du = y\,dy$

$= \frac{3}{4}\int u^{-1/2}\,du = \frac{3}{4}\left(\frac{u^{1/2}}{1/2}\right)+c$

$= \frac{3}{4}(2)\sqrt{2y^2+5}+c$

$= \boxed{\frac{3}{2}\sqrt{2y^2+5}+c}$

Function $f(x)$	Antiderivative $F(x)$
1	$x + c$
$2x$	$x^2 + c$
x^3	$\frac{1}{4}x^4 + c$
$\cos x$	$\sin x + c$
$\sin 2x$	$-\frac{1}{2}\cos 2x + c$

$F'(x) = f(x)$

$\int f(x)\,dx = F(x) + c$

$\longrightarrow D_x\left(-\frac{1}{2}\cos(2x)\right)$

$= -\frac{1}{2}(-\sin(2x))(2)$

$= \sin(2x)$