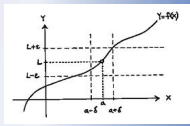
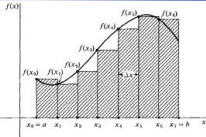


## 22 Antiderivatives



$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$\frac{d}{dx} \int_a^x f(t) dt = f(x)$$



$$\lim_{\max \Delta x_i \rightarrow 0} \sum_{i=1}^n f(x_i) \Delta x_i = \int_a^b f(x) dx$$

$$\int_a^b f(x) dx = F(b) - F(a)$$

# Antiderivatives

Function $f(x)$	Antiderivative $F(x)$
1	$x$
$2x$	$x^2$
$x^3$	$\frac{1}{4}x^4$
$\cos x$	$\sin x$
$\sin 2x$	$-\frac{1}{2} \cos 2x$

### Definition: Antiderivative

We call  $F$  an antiderivative of  $f$  on the interval,  $I$ , if

$$D_x F(x) = f(x) \text{ on } I.$$

ie. If  $F'(x) = f(x)$  for all  $x$  on the interval.

### Power Rule Theorem

For every real value of  $r$  except  $r = -1$ , then

$$\int x^r dx = \frac{x^{r+1}}{r+1} + C$$

Indefinite Integral is a linear operator.

## 22 Antiderivatives

EX 1 Evaluate the following integrals.

a)  $\int(2x^4 + 3x^2 - 7)dx$

b)  $\int(u^3 - u^9)du$

EX 2 Evaluate the following integrals.

a)  $\int\left(\frac{1}{y^2} + y^{\frac{1}{3}}\right)dy$

b)  $\int\left(x^{-4} + \sqrt[3]{x^2} - \frac{3}{x^5}\right)dx$

## 22 Antiderivatives

### Theorem

$$\int \sin x \, dx = -\cos x + C$$

$$\int \cos x \, dx = \sin x + C$$

$$\text{EX 3} \quad \int (t^2 - 2\cos t) \, dt$$

### Generalized Theorem

Let  $g$  be differentiable and  $r$  a rational number,  $r \neq -1$ , then

$$\int [g(x)]^r g'(x) \, dx = \frac{[g(x)]^{r+1}}{r+1} + C$$

$$\text{EX 4} \quad \int (4x^3 + 1)^4 12x^2 \, dx$$

## 22 Antiderivatives

EX 5  $\int (5x^2 + 1)\sqrt{5x^3 + 3x - 2} dx$

EX 6  $\int \frac{3y}{\sqrt{2y^2 + 5}} dy$

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